NAPREDNE METODE TEHNOLOGIJE PLASTIČNOG DEFORMISANJA

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Uvodne napomene

Predavanja

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Vežbe

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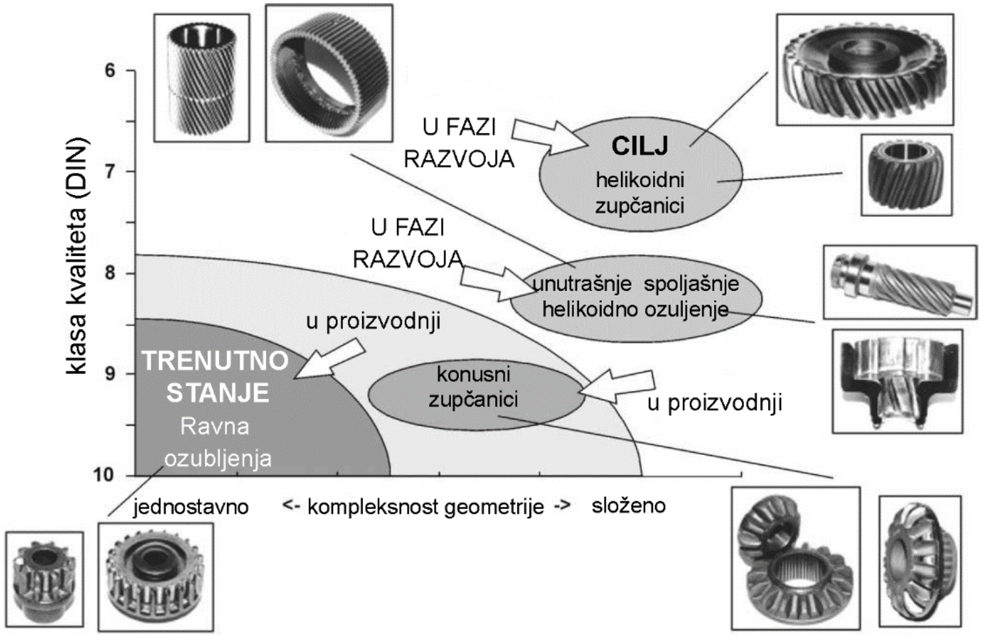
Literatura:

1. Knjiga - Mladomir Milutinović, Milija Kraišnik: Nekonvencionalni postupci obrade plastičnim deformisanjem
2. Knjiga - Branislav Devedžić: Osnovi teorije plastičnog deformisanja materijala
3. Skripte
4. Ostali nastavni materijal

Nastavni materijal: http://www.dpm.ftn.uns.ac.rs/sr/studenti/nastavni-materijal/cetvrta-godina-oas/napredne-metode-tehnologije-plasticnog-deformisanja

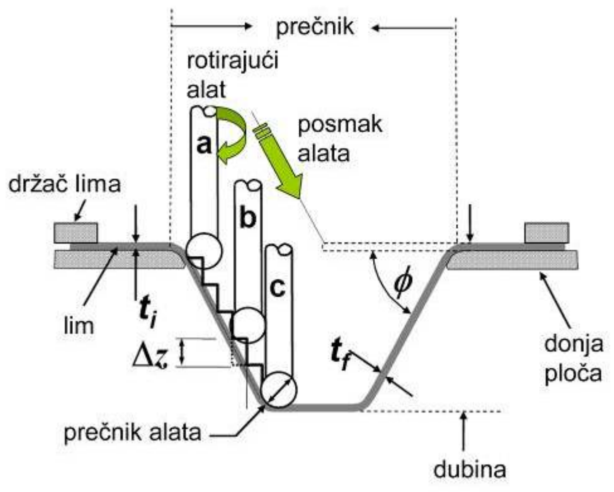
TRENDOVI RAZVOJA TPD

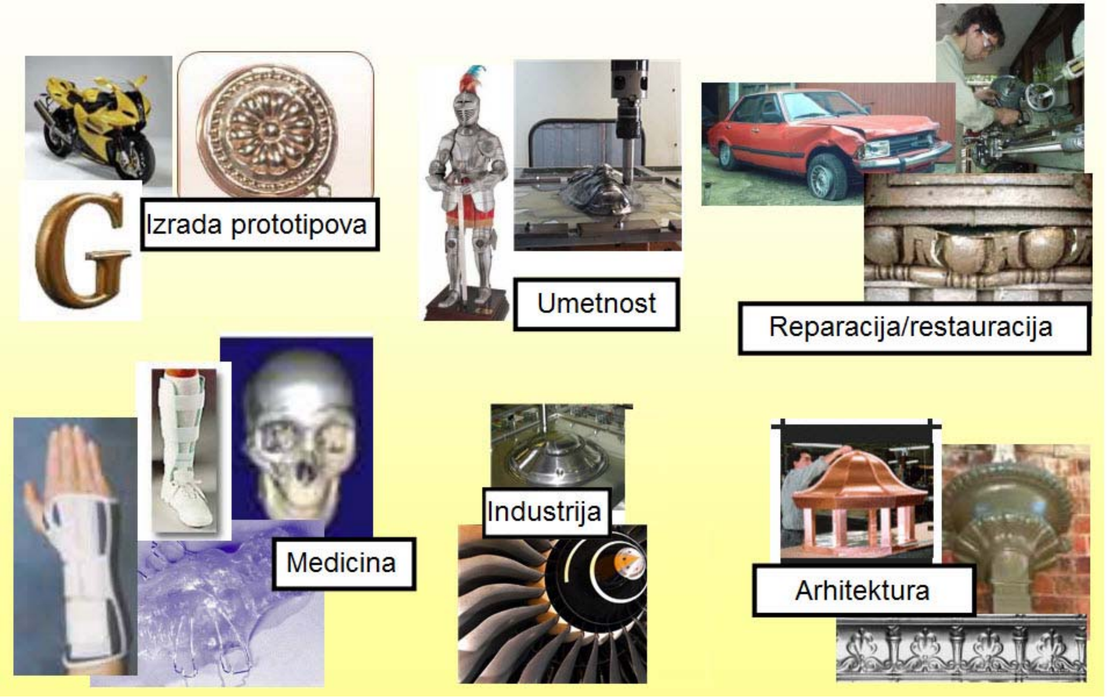


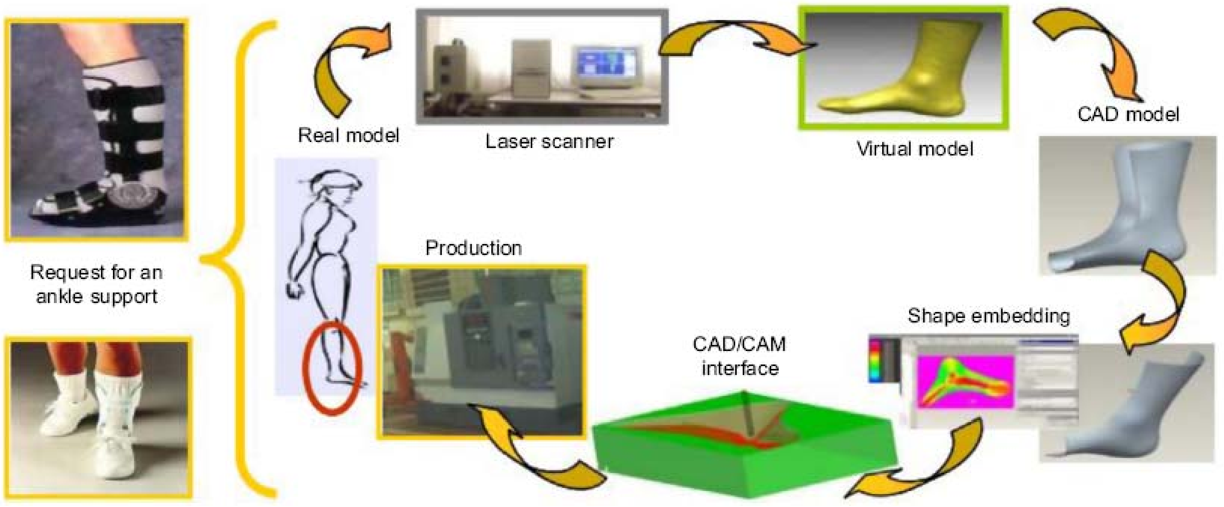
KONCEPT ODRŽIVE PROIZVODNJE!!!

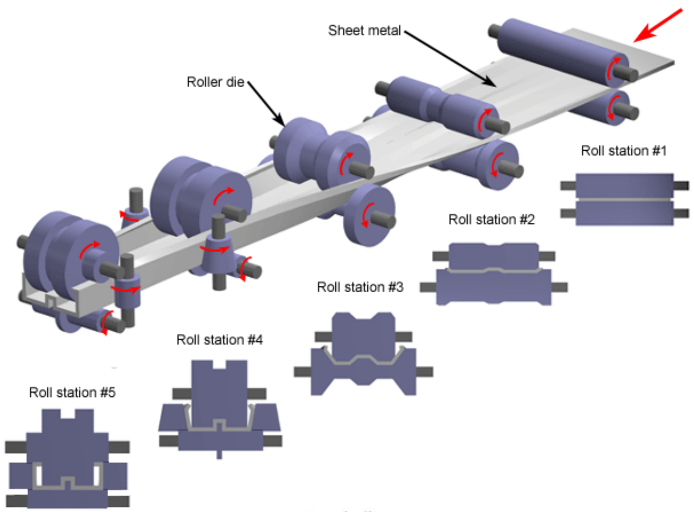
Hidroforming
Tailored blanks sheet forming
Inkrementalno deformisanje
Mikrodeformisanje
Net Shape Forming

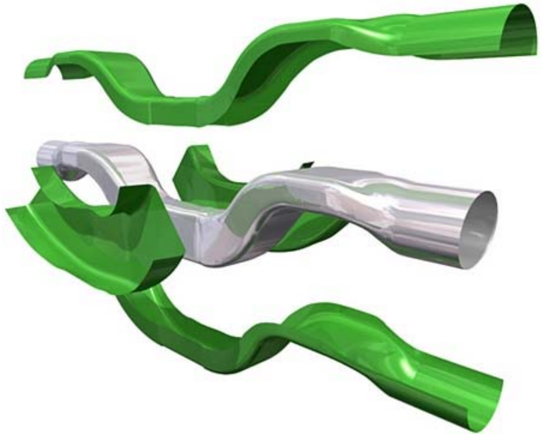
Inkrementalno deformisanje lima

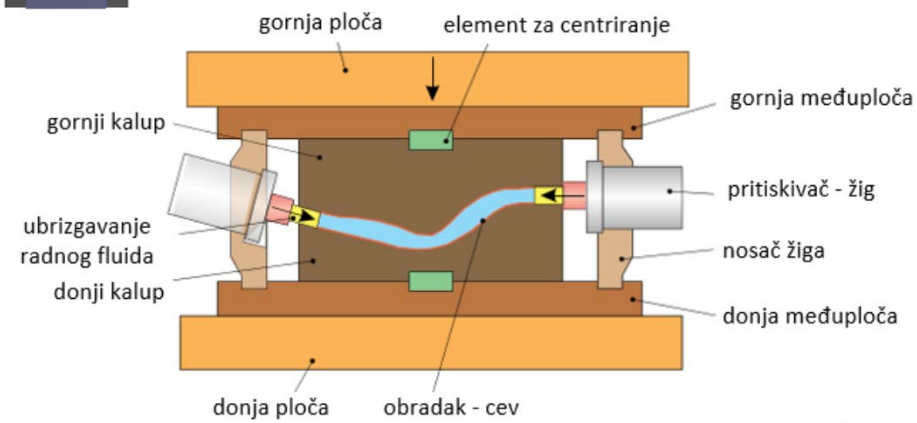


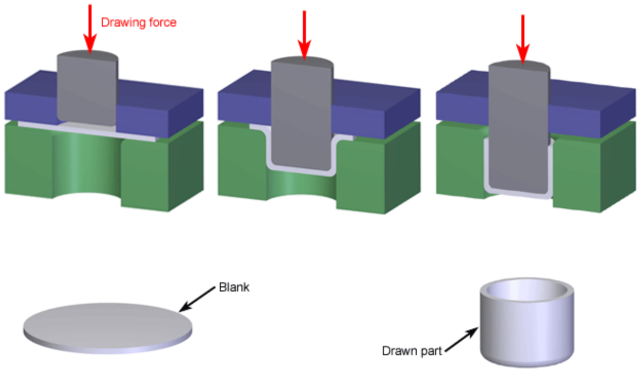


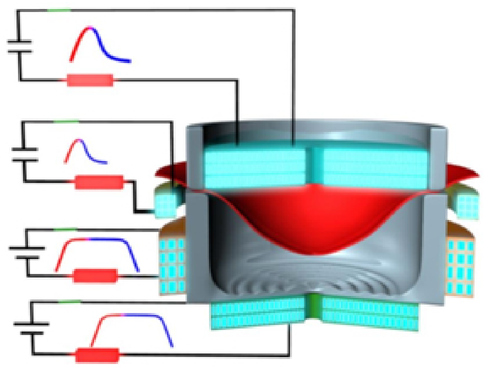



Sheet metal
Roller die
Roll station #1
Roll station #2
Roll station #3
Roll station #4
Roll station #5

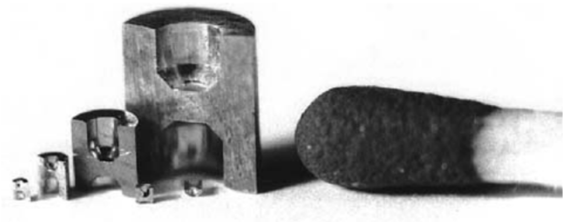


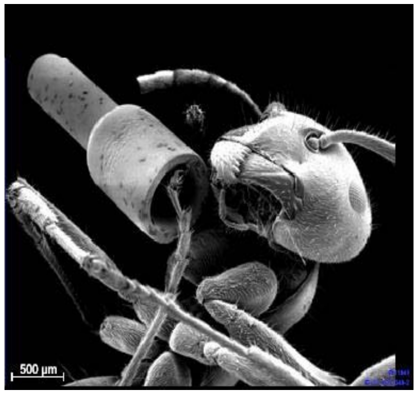

gornja ploča
element za centriranje
gornja međuploča
gornji kalup
pritiskivač - žig
ubrizgavanje
radnog fluida
nosač žiga
donji kalup
donja međuploča
donja ploča
obradak - cev

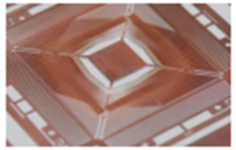
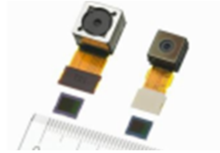
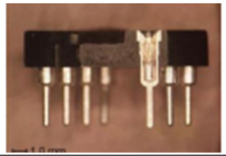
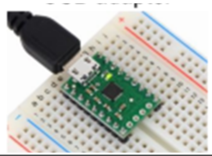
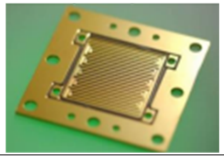
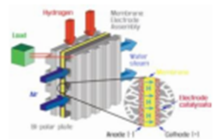
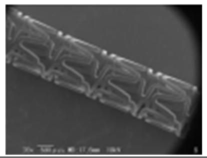
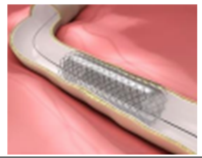

Drawing force
Blank
Drawn part



Mikrodeformisanje

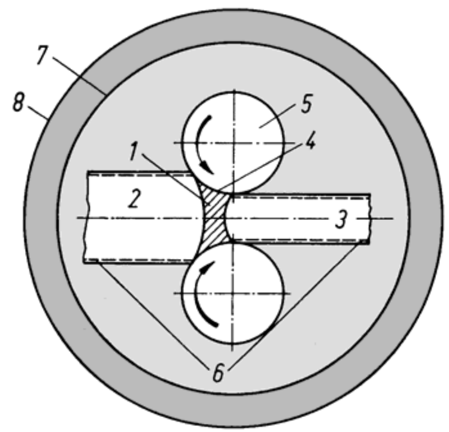


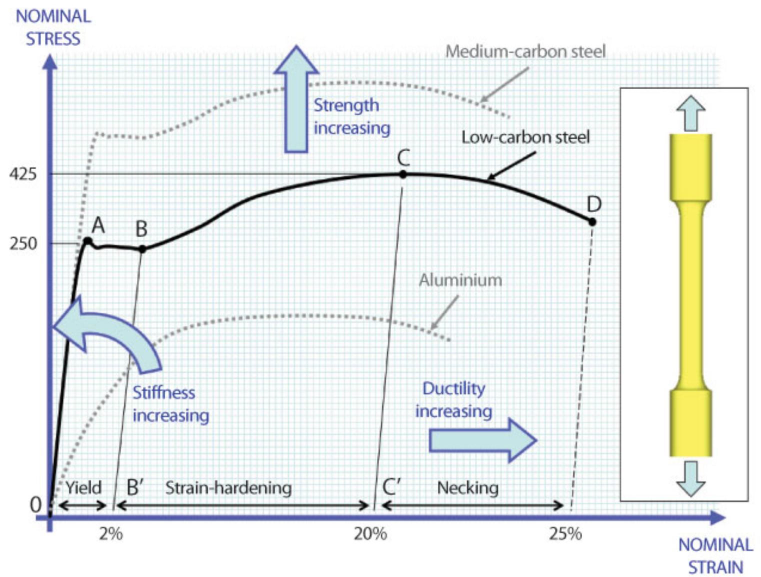


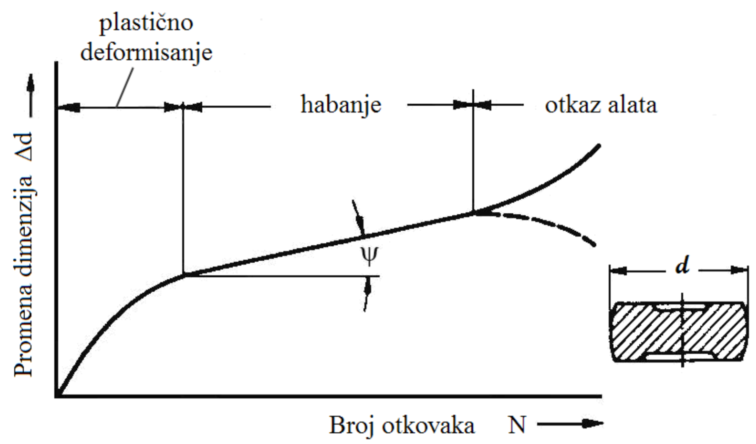
Proces	*Mikrokomponente*	*Aplikacija*
Mikroprosecanje	*Vodeći okvir* 	*Senzor slike* 
Mikroprosecanje *Mikrosavijanje*	*Magnetna glava diska* 	*Mikro HDD* 
Mikro duboko izvlačenje *Mikroprobijanje*	*Ploča mlaznice* 	*Injektor goriva* 
Mikroistiskivanje	*Mikročivije* 	*USB adapter* 
Mikroutiskivanje	*Bipolarna ploča* 	Goriva ćelija 
Mikro lasersko sečenje	*Stent* 	*Stent koronarne arterije* 

Problematika procesa **Obrade deformisanjem**

1. Zona deformisanja
2. Karakteristike materijala pre deformisanja
3. Karakteristike materijala nakon deformisanja
4. Kontakt alat–materijal i procesi koji se u tom kontaktu odvijaju za vreme procesa deformisanja
5. Problematika alata
6. Procesi koji se odvijaju između materijala i okoline pre i posle njegovog prolaska kroz zonu deformisanja
7. Mašina za TPD
8. Pogon u kome se proces izvodi, uključujući probleme unutrašnjeg transporta, automatizacije i dr.





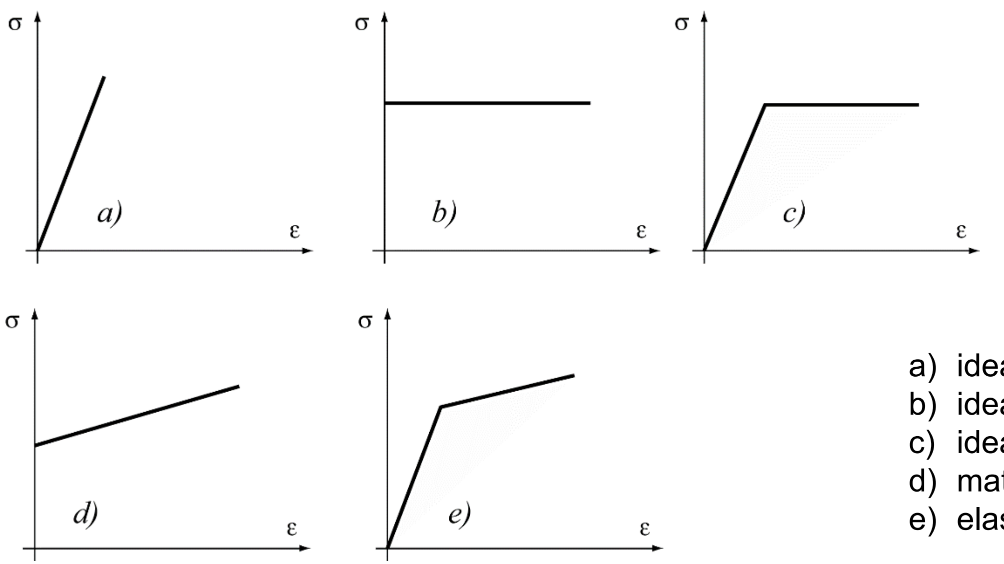


Teoretske osnove tehnologije plastičnog deformisanja

Parametri procesa deformisanja (naponi, deformacije, deformaciona sila i deformacioni rad)

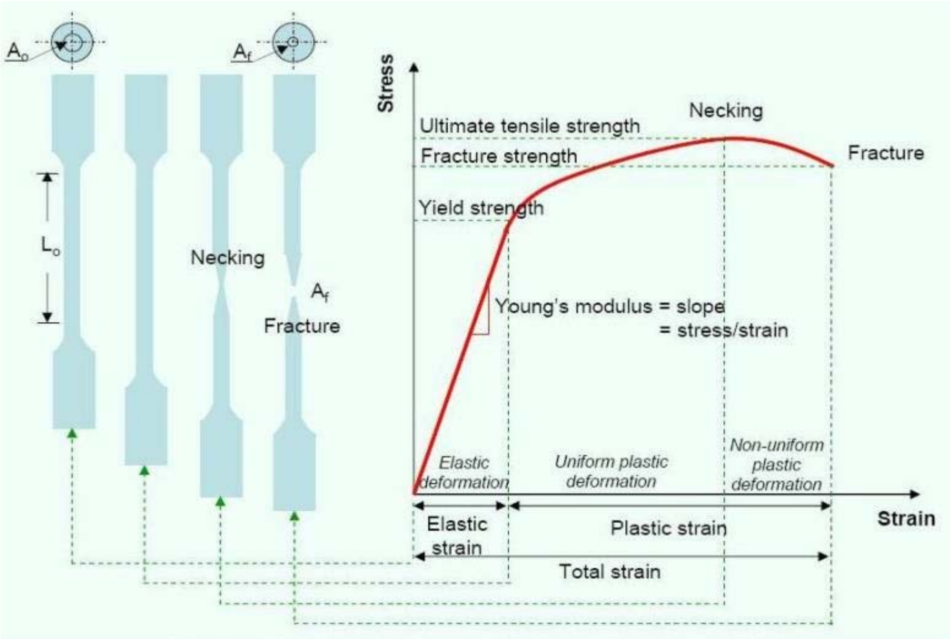
Osnovne hipoteze TPD:

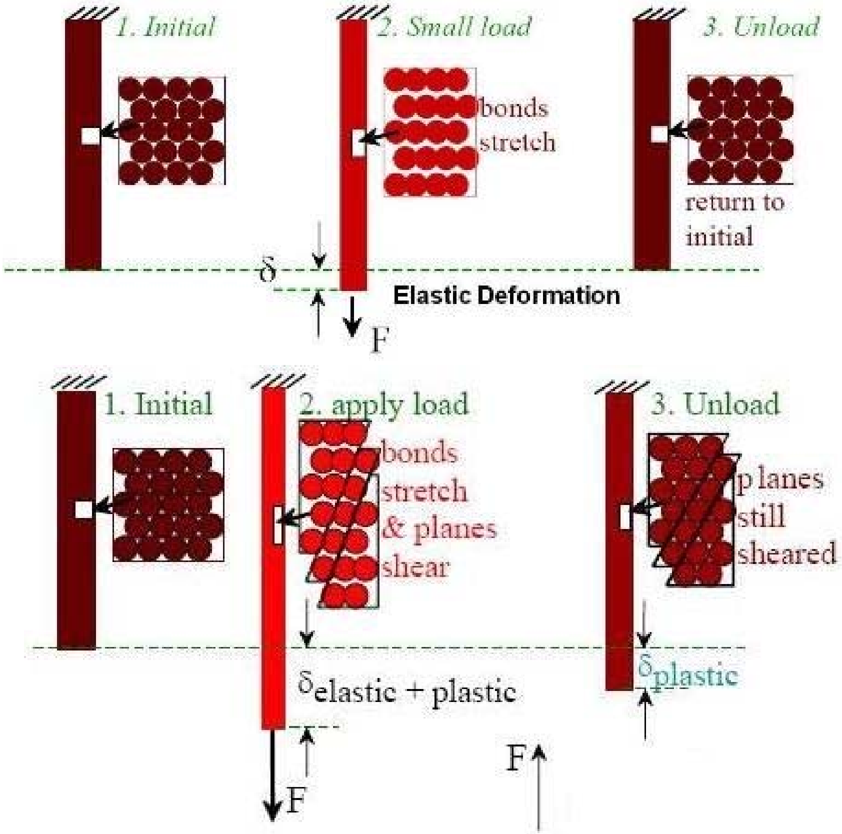
1. Hipoteza o homogenosti elastično-plastičnog tela
2. Hipoteza o prirodnom naponskom stanju
3. Hipoteza o izotropnosti strukture materijala
4. Hipoteza o idealizaciji elastičnih i plastičnih svojstava
5. Zanemarivanje elastičnih deformacija – kruta plastičnost
6. Konstantnost zapremine



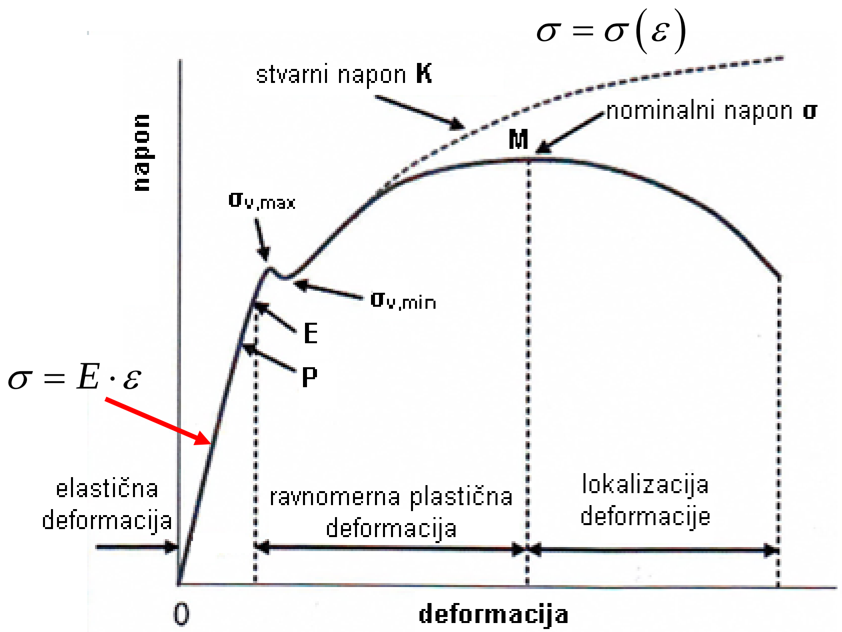
a) idealno elastičan materijal,
b) idealno plastičan materijal
c) idealno elastično-plastičan materijal
d) materijal s linearnim ojačavanjem
e) elastično-plastično telo s linearnim ojačavanjem

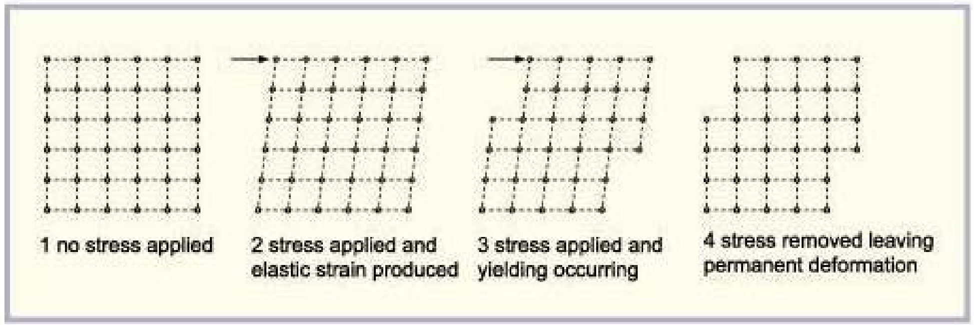
Elastične i plastične deformacije





Schematic Representation of Plastic Deformation

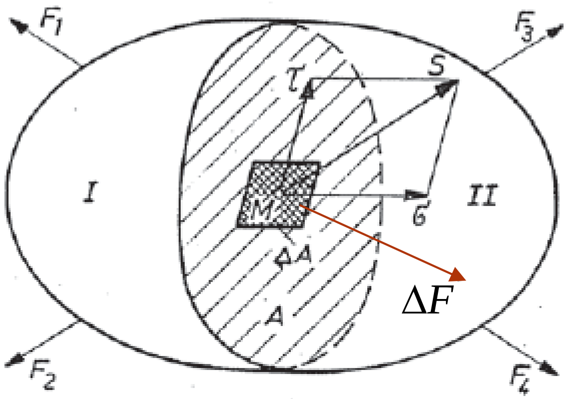




Stages of Plastic Deformation

Pojam i definicija napona

Napon - mera prosečne sile (ΔF) po jedinici površine (ΔA)

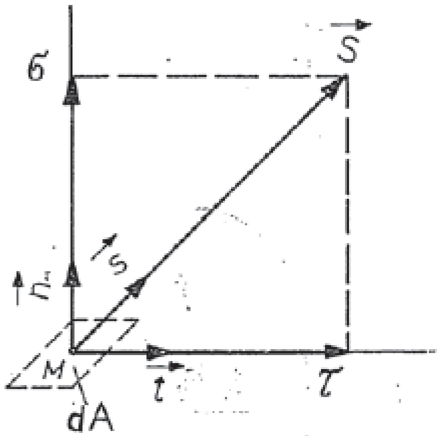


$$S = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

$$S^2 = \sigma^2 + \tau^2$$

σ - normalni napon
τ - tangencijalni napon

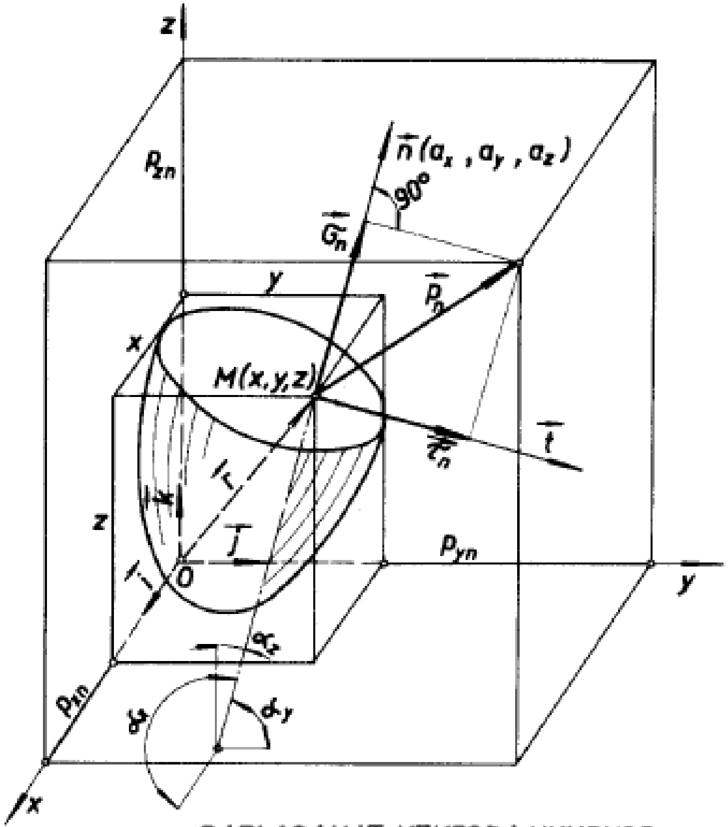
Napon je vektorska fizička veličina koja opisuje unutrašnje stanje napregnutog tela - zavisi od lokacije u telu i orijentacije ravni na kojoj deluje sila.



$$\vec{S} = \lim_{\Delta A \to 0} \frac{\Delta \vec{F}}{\Delta A}$$

$$\vec{S} = \sigma\,\vec{n} + \tau\,\vec{i}$$

Pojam i definicija napona

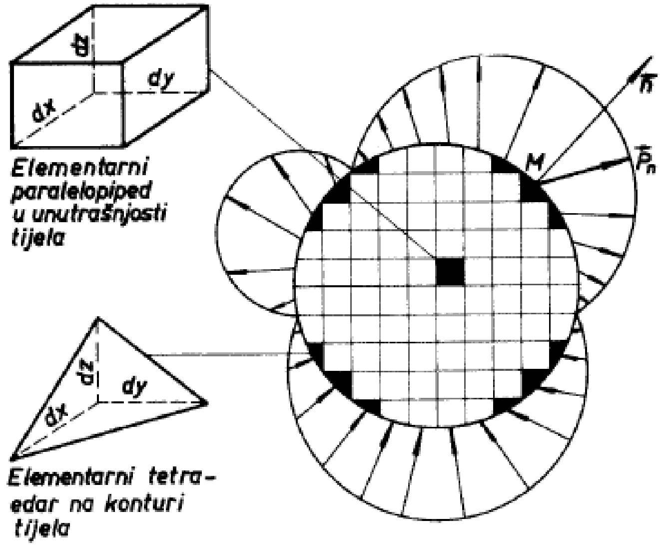


RAZLAGANJE VEKTORA UKUPNOG NAPONA

$$\vec{p}_n = p_{xn}\vec{i} + p_{yn}\vec{j} + p_{zn}\vec{k}$$ - po osama

$$\vec{p}_n = \sigma_n \vec{n} + \tau_n \vec{t}$$ - po pravcima

$$p_n = |\vec{p}_n| = \sqrt{\sigma_n^2 + \tau_n^2}$$



$$\vec{r} = \vec{r}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$$

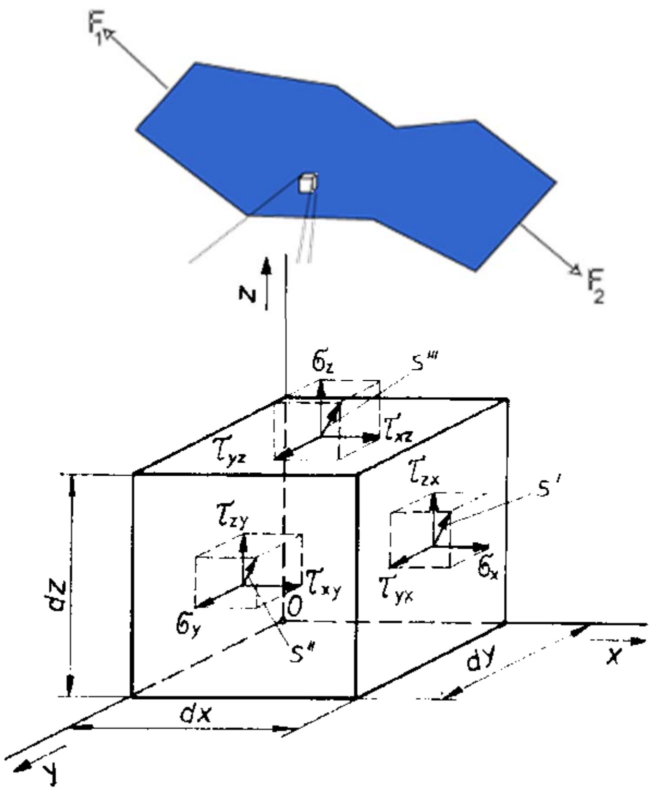
$$\vec{n} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\left.\begin{aligned} a_x &= \cos\alpha_x = \vec{n}\cdot\vec{i} \\ a_y &= \cos\alpha_y = \vec{n}\cdot\vec{j} \\ a_z &= \cos\alpha_z = \vec{n}\cdot\vec{k} \end{aligned}\right\}$$

Naponsko stanje. Tenzor napona

Naponsko stanje - stanje tela izloženog dejstvu spoljašnjih sila (površinske i zapreminske) i u kome je uspostavljena unutrašnja ravnoteža elastičnih veza između čestica materijala.

- Naposko stanje je poznato ako su poznati normalni i tangencijalni (smičući) napon u ma kom pravcu, odnosno za ma kako orijentisanu elementarnu površinu.
- Naponsko stanje je tenzorska veličina!!!!



σ_x	τ_{xy}	τ_{xz}	pravac ose x
τ_{yx}	σ_y	τ_{yz}	pravac ose y
τ_{zx}	τ_{zy}	σ_z	pravac ose z
ravan upravna na x	ravan upravna na y	ravan upravna na z	

$$T_\sigma = \begin{Vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{Vmatrix}$$

$$\sum M_z = 0$$

$$\left(\tau_{xy} dxdy\right) dy - \left(\tau_{yx} dydz\right) dx = 0$$

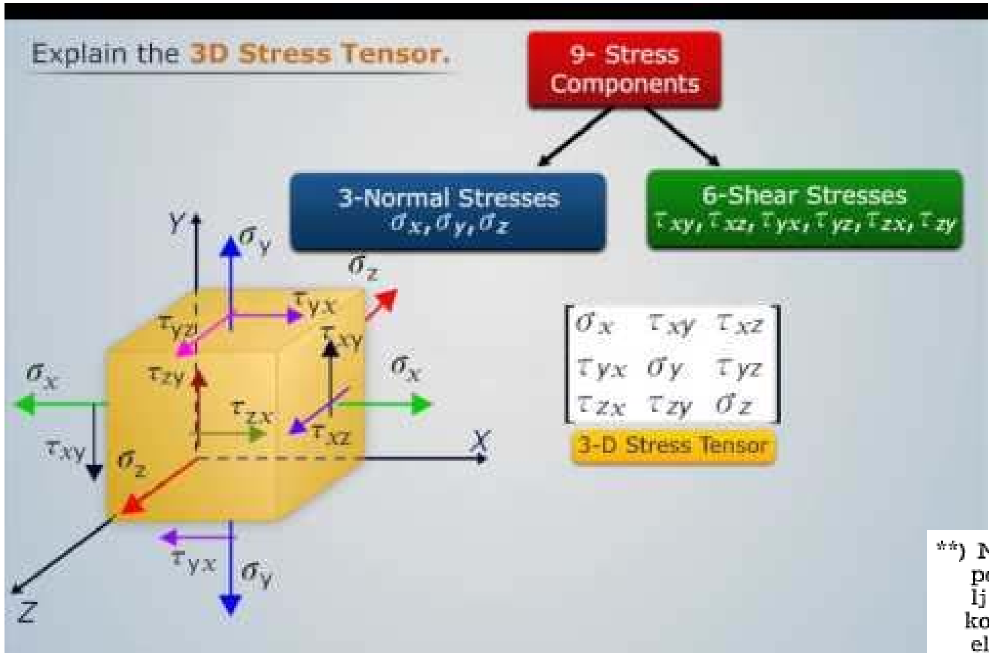
$$\tau_{xy} = \tau_{yx}$$

$$\therefore$$

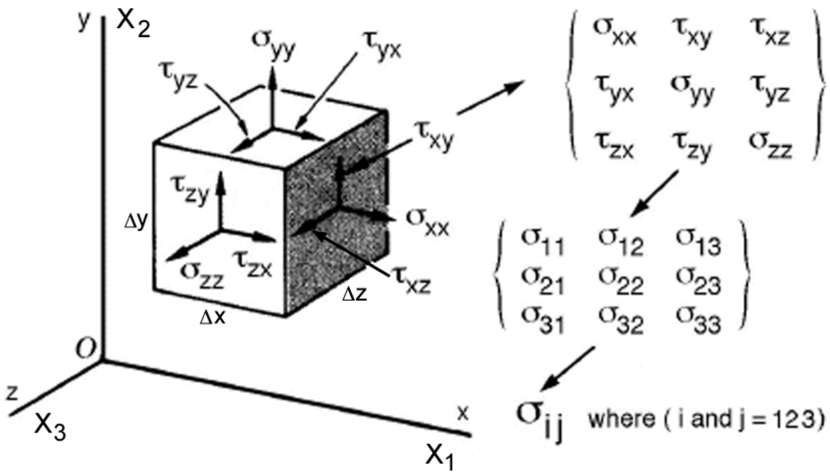
$$\tau_{xz} = \tau_{zx}$$

$$\tau_{yz} = \tau_{zy}$$

Naponsko stanje. Tenzor napona



Tenzor (grč. tensio što znači naprezanje) je vektor određenog vektorskog prostora i kao matematička struktura predstavlja uopštenje vektora. Tenzorske veličine su fizičke veličine čija vrednost zavisi i od **koordinate!!!!!** One se matematički predstavljaju matricom.



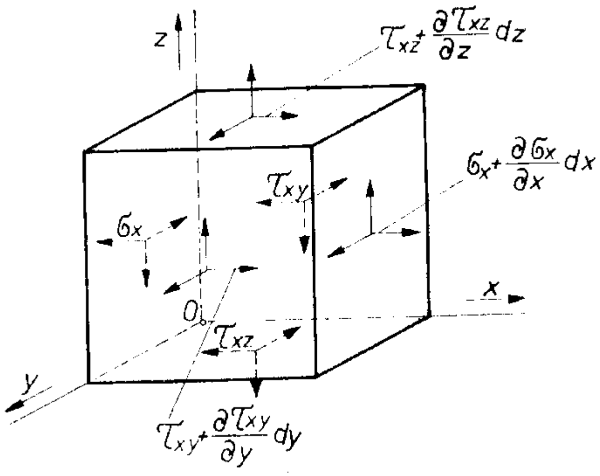
**) Ne ulazeći ovde dublje u suštinu pojma i svojstava tenzora, može se pojednostavljeno reći da je uobičajeno da se tenzor napona predstavlja kao šematski (matrični) skup svih normalnih i smičućih napona koji deluju na tri uzajamno upravne površine beskonačno malog elementa zapremine. Pri tome su svi naponi (komponente tenzora) raspoređeni po određenom redosledu (sistemu). Tako oformljeni tenzori imaju svojstvo različitih matematičkih transformacija (mogu se sabirati, oduzimati, itd.) Treba istaći da se komponente tenzora pri promeni orijentacije koordinatnog sistema ne mogu menjati proizvoljno, već po određenoj zakonitosti, tako da se suštinska svojstva tenzora time ne menjaju (v. poglavlja o invarijantama). Zahvaljujući ovakvim, a i drugim osobenostima (o kojima će kasnije biti još reči), tenzori napona mogu izraziti suštinu naponskog stanja. Osim tenzora napona, u teoriji plastičnosti takođe se koriste tenzori deformacija i tenzori brzina deformacija.

Napomena:

u mehanici neprekidnih sredina često se ispisivanje simbola za napone unekoliko pojednostavljuje time što se i za normalne i za smičuće napone koristi simbol σ , a razlike između njih proizlaze iz korišćenih indeksa. Naime, za normalne napone bilo bi: σ_{xx} ; σ_{yy} ; σ_{zz} ; a za smičuće: σ_{xy} ; σ_{yz} ; σ_{zx} , tako da bi opšta oznaka napona bila $\sigma_{ij} = \sigma_{ji}$, a ona se takođe može koristiti i kao opšti skraćeni simbol za tenzor napona ($T_\sigma = \sigma_{ij}$). što pojednostavljuje ispisivanje.

Diferencijalne jednačine ravnoteže

Veličina napona u opštem slučaju menja se od tačke do tačke tela i predstavlja neprekidnu funkciju koordinata!!!
Potrebno je pronaći raspored po čitavom telu!?? – moguće/nemoguće



$$\sum F_x = 0$$

$$\left(\sigma_x + \frac{\partial \sigma_x}{\partial x}dxdy\right)dydz - \sigma_x dydz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y}dy\right)dxdz -$$

$$-\left(\tau_{xy}dxdz\right) + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial z}dz\right)dxdy - \left(\tau_{xy}dxdy\right) = 0$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0$$

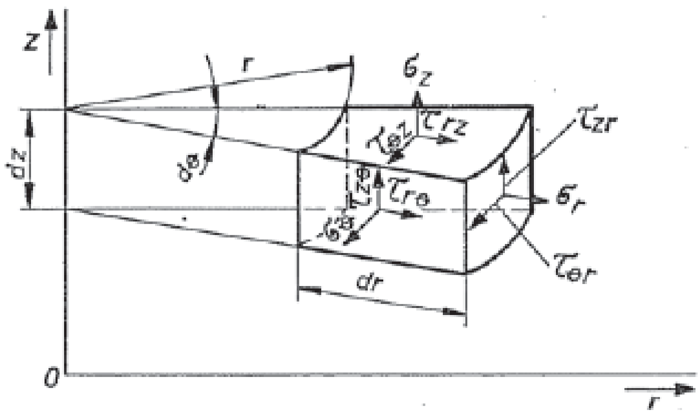
$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

Za rešavanje gornjeg sistema jednačina granični uslovi moraju biti poznati!!!!!

Diferencijalne jednačine ravnoteže

Cilindrični koordinatni sistem (r, ϴ, z)

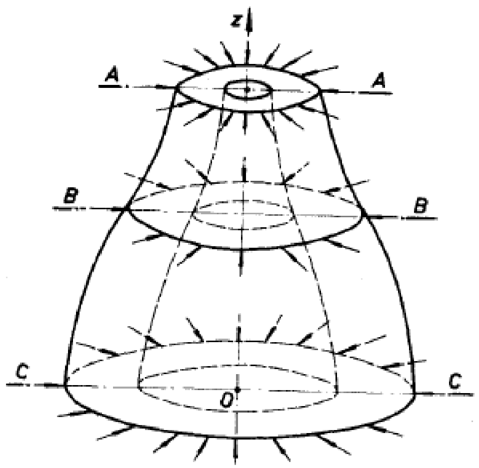


$$\frac{\partial \sigma_r}{\partial r}+\frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta}+\frac{\partial \tau_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}=0$$

$$\frac{\partial \tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_{\theta}}{\partial \theta}+\frac{\partial \tau_{\theta z}}{\partial z}+\frac{2\tau_{r\theta}}{r}=0$$

$$\frac{\partial \tau_{rz}}{\partial r}+\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}+\frac{\partial \sigma_z}{\partial z}+\frac{\tau_{rz}}{r}=0$$

$$T_\sigma = \begin{Vmatrix} \sigma_r & \tau_{r\theta} & \tau r \\ \tau_{\theta r} & \sigma_\theta & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \sigma_z \end{Vmatrix}$$



Osno-simetrično naponsko stanje($\tau_{r\Theta}=\tau_{z\Theta}=0$)

- Naponi ne zavise od komponente ϴ!!!

$$T_\sigma = \begin{Vmatrix} \sigma_r & 0 & \tau r \\ 0 & \sigma_\theta & 0 \\ \tau_{zr} & 0 & \sigma_z \end{Vmatrix}$$

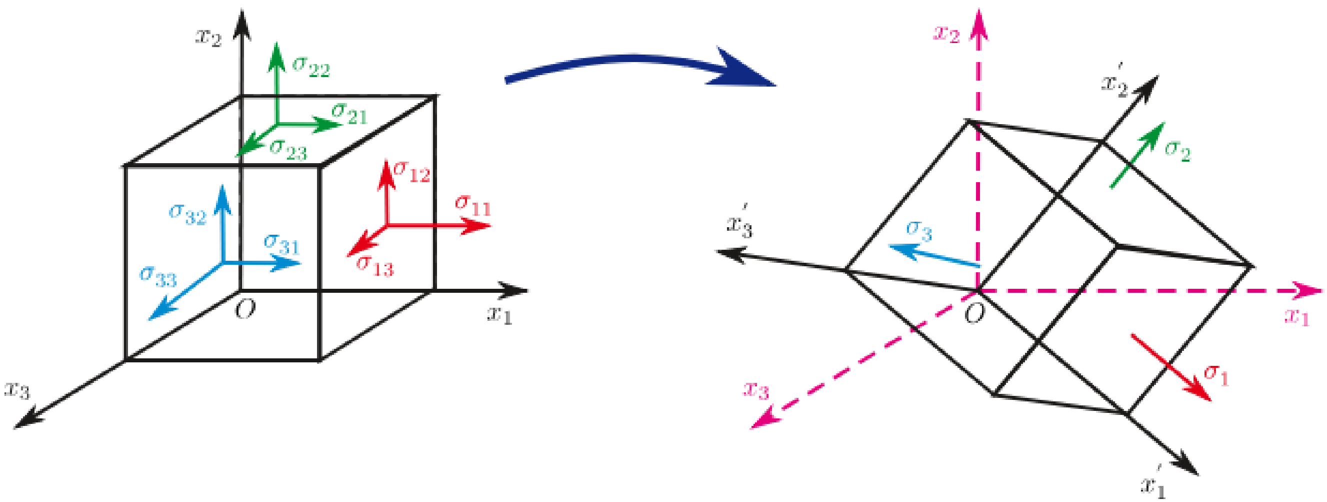
Glavni normalni naponi

Naponi u ravnuima u kojima nema smičućih (tangencijalnih) napona (τ=0)

Naposnsko stanje u tački je određeno (poznato) ako su poznati glavni naponi i pravci glavnih osa!!!!

$$T_\sigma = \begin{Vmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{Vmatrix}$$

$$\boxed{\sigma_1 \geq \sigma_2 \geq \sigma_3}$$

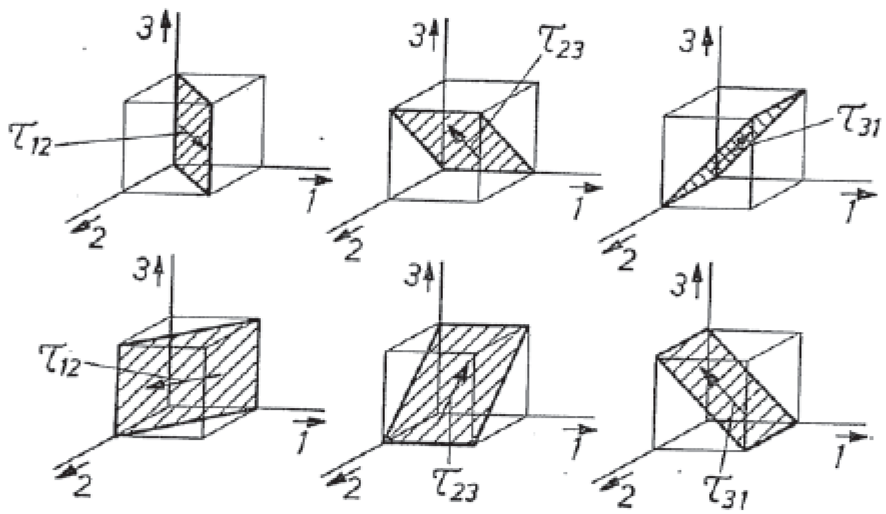


U opštem slučaju pravci glavnih osa su različiti u različitim tačkama!
U većini slučajeva obrade deformisanjem se smatra da su pravci glavnih osa isti (približno) u svim tačkama (pojednostavljenje)!!!!!!

Glavni smičući (tangencijalni naponi)

Smičući (tangencijalni) naponi maksimalni ($\tau = \tau_{max}$)

Deluju u ravnima od kojih je svaka upravna na jednu glavnu koordinatnu ravan (osu) a sa ostale dve ravni (ose) zaklapa ugao od 45°!!!



$$\tau_{12} = \pm\frac{1}{2}(\sigma_1 - \sigma_2) = 0$$

$$\tau_{23} = \pm\frac{1}{2}(\sigma_2 - \sigma_3) = 0$$

$$\tau_{31} = \pm\frac{1}{2}(\sigma_3 - \sigma_1) = 0$$

$$\tau_{12} + \tau_{23} + \tau_{31} = 0$$

Iz uslova $\sigma_x \geq \sigma_y \geq \sigma_z$ sledi $\tau_{31} = \tau_{max}$

$$\sigma_{12} = \frac{\sigma_1 + \sigma_2}{2}$$

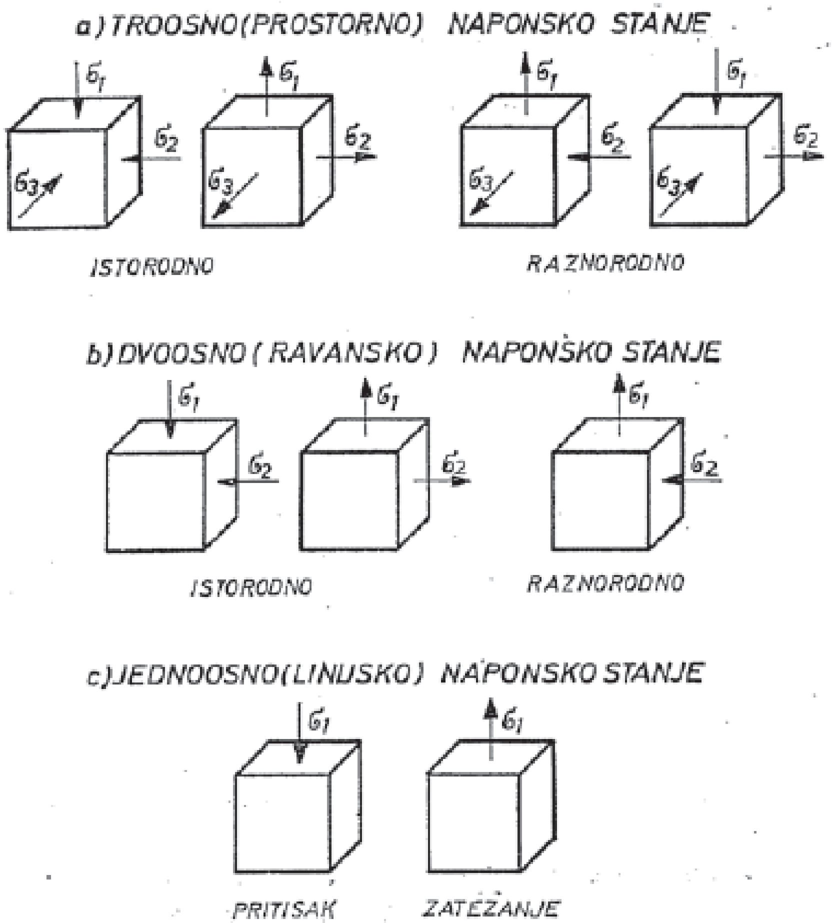
$$\sigma_{23} = \frac{\sigma_2 + \sigma_3}{2}$$

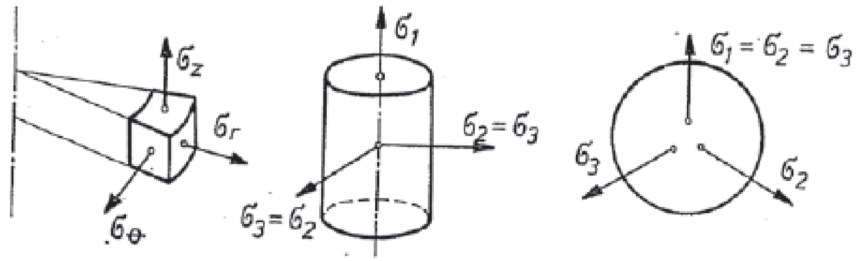
$$\sigma_{31} = \frac{\sigma_3 + \sigma_1}{2}$$

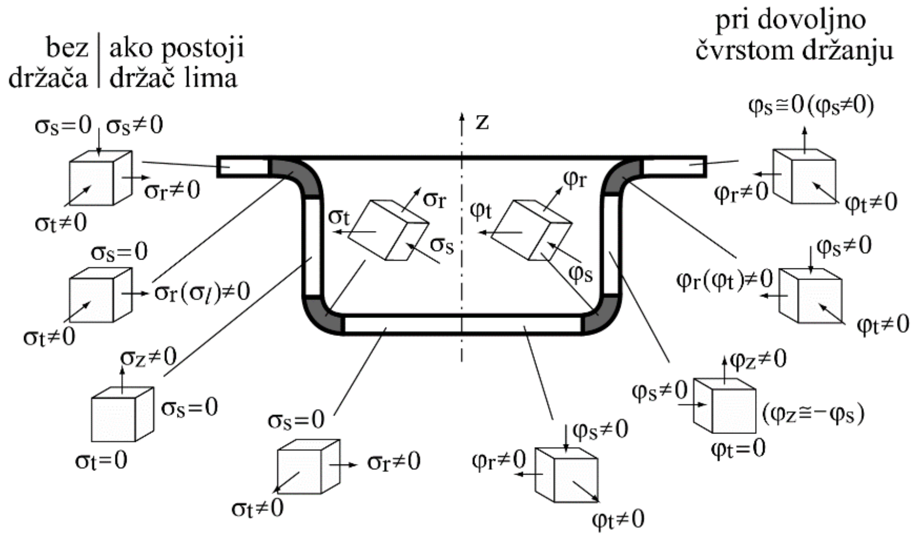
Vredností normalnih napona u ravnima gde dejstvuju glavni smičući naponi!

Vrste naponskih stanja i njihove mehaničke šeme

Naponske šeme prikazuju pravce i smerove glavnih normalnih napona u posmatranom procesu.
Uticaj na deformabilnost materijala.

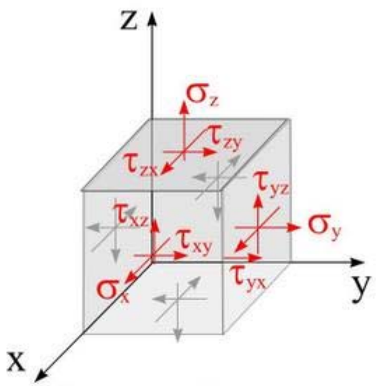




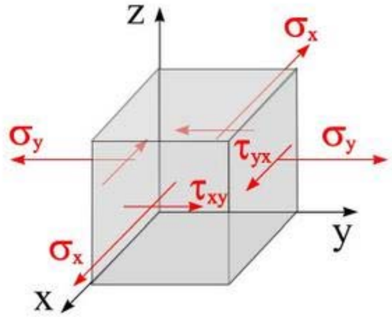


Vrste naponskih stanja i njihove mehaničke šeme

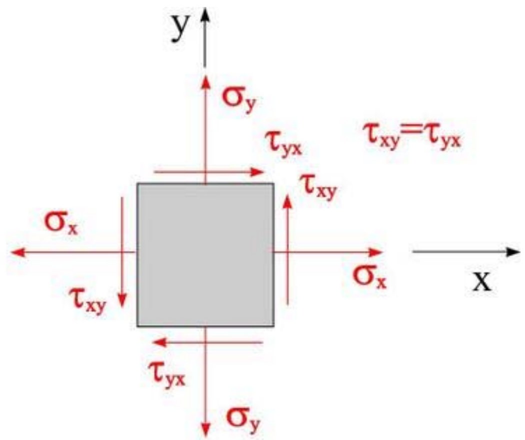
Specijalni slučajevi naposkog stanja
- Ravansko naponsko stanje ($\sigma_x \neq 0$, $\sigma_y \neq 0$, $\sigma_z = 0$; $\varepsilon_x \neq 0$, $\varepsilon_y \neq 0$, $\varepsilon_z \neq 0$)
- Ravansko deformaciono stanje ($\sigma_x \neq 0$, $\sigma_y \neq 0$, $\sigma_z \neq 0$; $\varepsilon_x \neq 0$, $\varepsilon_y \neq 0$, $\varepsilon_z = 0$)



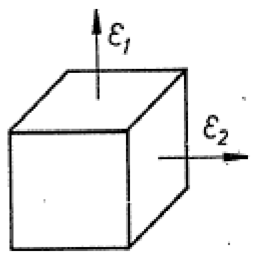
Prostorno stanje napona



Ravno stanje napona



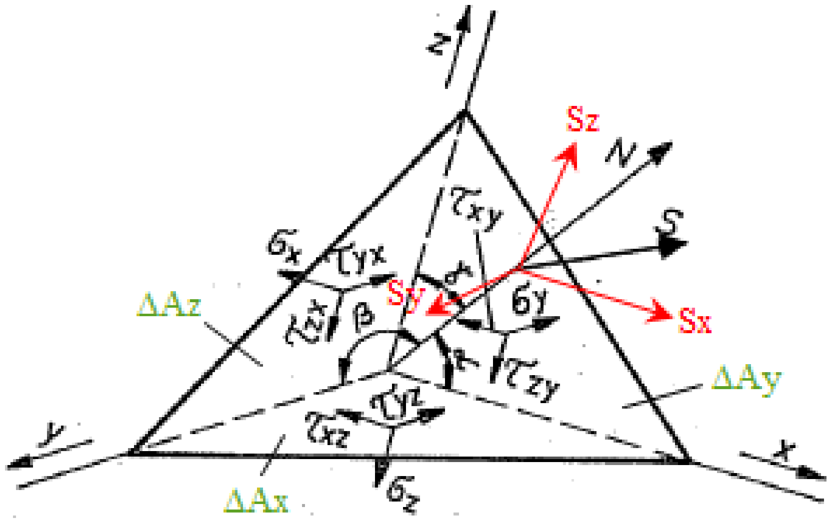
Ravno stanje napona (dvodimenzionalni prikaz)



Ravansko deformaciono stanje

Naponi u kosoj ravni

Naponi u tri međusobno upravne ravni koje prolaze kroz jednu tačku u potpunosti definišu naponsko stanje.
Ako su poznati naponi u te tri ravni onda se mogu odrediti naponi u ma kojoj ravni (proizvojno orijentisanoj) koja prolazi kroz tu tačku.



$$\cos\alpha = \cos(N,x) = \alpha_x$$
$$\cos\beta = \cos(N,y) = \alpha_y$$
$$\cos\gamma = \cos(N,z) = \alpha_z$$
$$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$$

Uglovi (kosinusi uglova) koje nagnuta ravan zaklapa sa koordinatnim osama

$$S_x\Delta A - \sigma_x\Delta A_x - \tau_{xy}\Delta A_y - \tau_{xz}\Delta A_z = 0$$
$$S_y\Delta A - \tau_{yx}\Delta A_x - \sigma_y\Delta A_y - \tau_{yz}\Delta A_z = 0$$
$$S_z\Delta A - \tau_{zx}\Delta A_x - \tau_z y\Delta A_y - \sigma_z\Delta A_z = 0$$
$$\Delta A_x = \Delta A\alpha_x; \quad \Delta A_y = \Delta A\alpha_y; \quad \Delta A_z = \Delta A\alpha_z$$

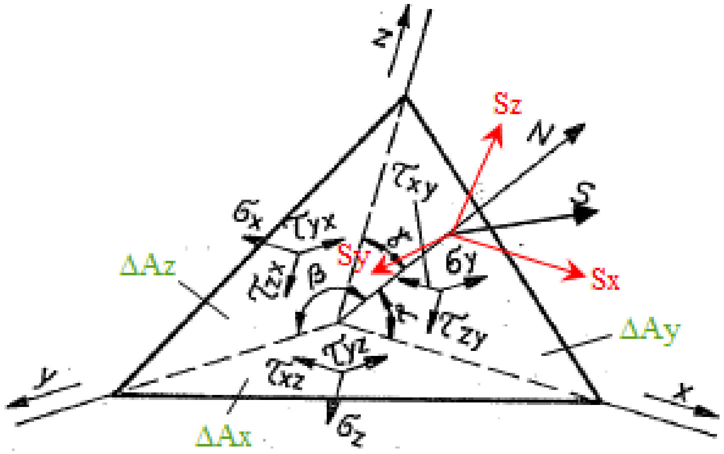
Košijeve (Cauchy) jednačine (konturni uslovi)

$$\boxed{\begin{aligned} S_x &= \sigma_x\alpha_x + \tau_{xy}\alpha_y + \tau_{xz}\alpha_z \\ S_y &= \tau_{yx}\alpha_x + \sigma_y\alpha_y + \tau_{yz}\alpha_z \\ S_z &= \tau_{zx}\alpha_x + \tau_{zy}\alpha_y + \sigma_z\alpha_z \end{aligned}}$$

Naponi u kosoj ravni

Ogisten Luj Koši	
	
Ogisten Luj Koši	
Datum rođenja	21. avgust 1789.
Mesto rođenja	Pariz Francuska
Datum smrti	23. maj 1857. (67 god.)

Košijeve (Cauchy) jednačine (konturni uslovi)



$$S_x = \sigma_x \alpha_x + \tau_{xy} \alpha_y + \tau_{xz} \alpha_z$$

$$S_y = \tau_{yx} \alpha_x + \sigma_y \alpha_y + \tau_{yz} \alpha_z$$

$$S_z = \tau_{zx} \alpha_x + \tau_{zy} \alpha_y + \sigma_z \alpha_z$$

$$\boxed{S^2 = S_x^2 + S_y^2 + S_z^2}$$

Napomena: s obzirom da koeficijenti pravaca u ovim jednačinama mogu imati najrazličitije iznose, to proizlazi da nagnuta površina dobija u opštem slučaju najrazličitije orijentacije. Zato se može takođe smatrati da ove jednačine predstavljaju na izvestan način vezu između unutrašnjih napona i onih na površini — tzv. konturni uslovi (granični uslovi na konturi), koji se mogu koristiti kao granični uslovi pri integrisanju diferencijalnih jednačina ravnoteže.

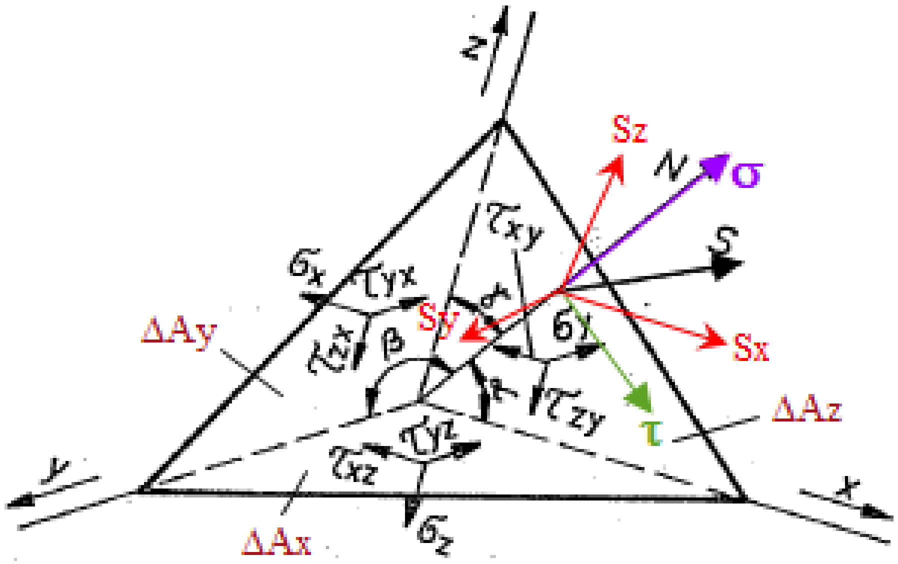
Naponi u kosoj ravni

Naponsko stanje u kosoj ravni izraženo preko **glavnih napona**.

- Projekcije ukupnog napona na glavne ose (S_1, S_2, S_3) i ukupni napon (S)

$$S_1 = \sigma_1 \alpha_1 \quad S_2 = \sigma_2 \alpha_2 \quad S_3 = \sigma_3 \alpha_3$$

$$S^2 = \sigma_1^2 \alpha_1^2 + \sigma_2^2 \alpha_2^2 + \sigma_3^2 \alpha_3^2$$



Normalni napon u kosoj ravni

$$\sigma = S_x \alpha_x + S_y \alpha_y + S_y \alpha_y$$

$$\boxed{\begin{aligned}\sigma &= \sigma_x \alpha_x^2 + + \sigma_y \alpha_y^2 + \sigma_z \alpha_z^2 + \\ &+ 2\tau_{xy} \alpha_x \alpha_y + \tau_{yz} \alpha_y \alpha_z + \tau_{zx} \alpha_z \alpha_x\end{aligned}}$$

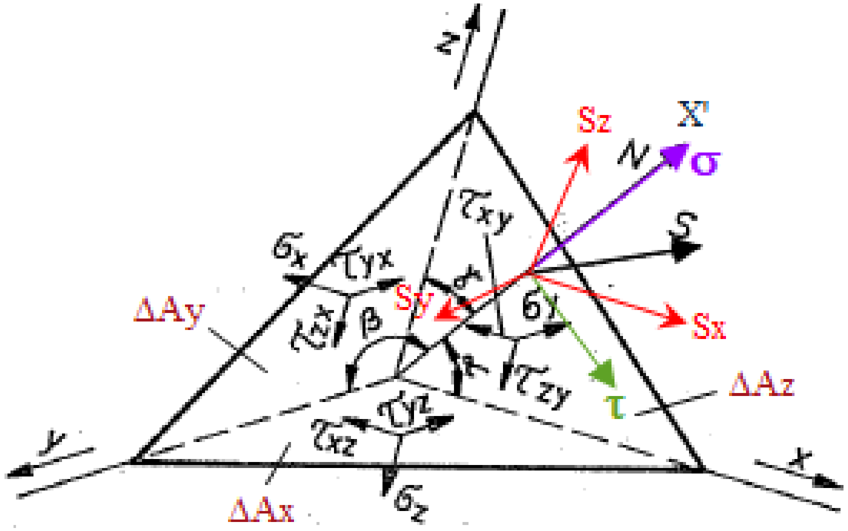
$$\boxed{\sigma = \sigma_1 \alpha_1^2 + \sigma_2 \alpha_2^2 + \sigma_3 \alpha_3^2}$$

Smičući napon u kosoj ravni

$$\boxed{\tau = \sqrt{S^2 - \sigma^2}}$$

Naponi u kosoj ravni

Rotacija koordinatnog sistema: (x, y, z) →(x`, y`, z`),  x` ≡N



normalni napon

$$\sigma = \sigma_{x'} = S_x \alpha_x + S_y \alpha_y + S_z \alpha_z$$

smičući napon

$$\tau_{x'y'} = S_x \cos(y', x) + S_y \cos(y', y) + S_z \cos(y', z)$$

$$\tau_{z'x'} = S_x \cos(z', x) + S_y \cos(z', y) + S_z \cos(z', z)$$

$$T_\sigma = \begin{Vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{Vmatrix} \qquad T_{\sigma'} = \begin{Vmatrix} \sigma_{x'} & \tau_{x'y'} & \tau_{x'z'} \\ \tau_{y'x'} & \sigma_{y}' & \tau_{y'z'} \\ \tau_{z'x'} & \tau_{z'y}' & \sigma_{z'} \end{Vmatrix}$$

isti tenzori!!!

VAŽNO:
rotacija (transformacija) koordinatnog sistema ne utiče. vrednost tenzora u posmatranoj tački!!!!

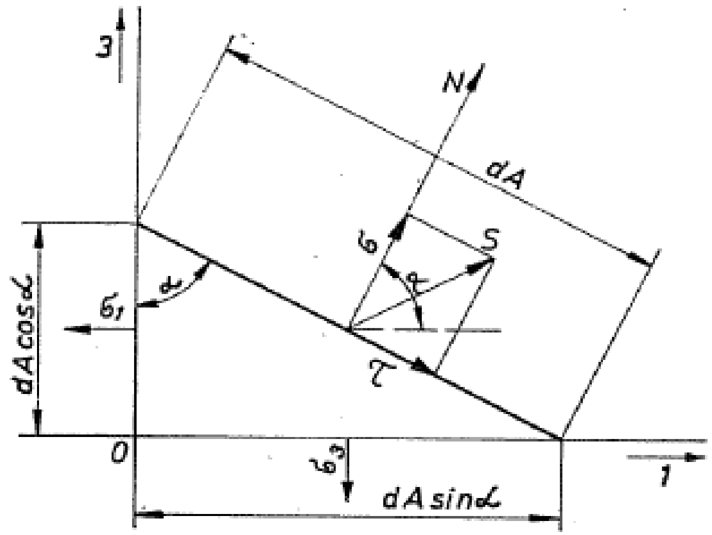
Naponi u kosoj ravni

Specijalni slučaj

Za početak plastičnog deformisanja bitne su vrednosti maksimalnog (σ_1) i minimalnog napona (σ_3) → ravansko naponsko stanje

$$\sigma \, \Delta A \cos\alpha - \sigma_1 \, \Delta A \cos\alpha + \tau \Delta A \sin\alpha = 0 \qquad \text{(za pravac 1)}$$

$$\sigma \, \Delta A \sin\alpha - \sigma_3 \, \Delta A \sin\alpha - \tau \Delta A \cos\alpha = 0 \qquad \text{(za pravac 3)}$$



$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \; ; \qquad \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\sin\alpha\cos\alpha = \frac{\sin 2\alpha}{2}$$

Normalni i smičući napon

$$\sigma = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\alpha$$

$$\tau = \pm\frac{\sigma_1-\sigma_3}{2}\sin 2\alpha$$

$\alpha=45° \rightarrow \tau=\tau_{max}$

$$\tau_{max} = \tau_{13} = \pm\frac{1}{2}(\sigma_1-\sigma_3) \qquad |\tau_{max}| = \frac{|\sigma_1-\sigma_3|}{2}$$

$$\sigma_{13} = \frac{\sigma_1+\sigma_3}{2}$$

Invarijante tenzora napona

Pretpostavka

Na nagnutnoj ravni deluju samo normalni naponi (S=σ) → glavna ravan

komponente napona:

$$S_x = \sigma a_x \;;\quad S_y = \sigma a_y \;;\quad S_z = \sigma a_z$$

Košijeve (Cauchy) jednačine

$$\begin{aligned}
\sigma a_x &= \sigma_x a_x + \tau_{xy} a_y + \tau_{xz} a_z \\
\sigma a_y &= \tau_{yx} a_x + \sigma_y a_y + \tau_{yz} a_z \\
\sigma a_z &= \tau_{zx} a_x + \tau_{zy} a_y + \sigma_z a_z
\end{aligned}$$

Sistem linearnih homogenih jednačina

$$\boxed{\begin{aligned}
(\sigma_x - \sigma) a_x + \tau_{xy} a_y + \tau_{xz} a_z &= 0 \\
\tau_{yx} a_x + (\sigma_y - \sigma) a_y + \tau_{yz} a_z &= 0 \\
\tau_{zx} a_x + \tau_{zy} a_y + (\sigma_z - \sigma) a_z &= 0
\end{aligned}}$$

trivijalno rešenje $a_x = a_y = a_z = 0$ nije moguće zbog $a_x^2 + a_y^2 + a_z^2 = 1$

$$\begin{vmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma \end{vmatrix} = 0$$

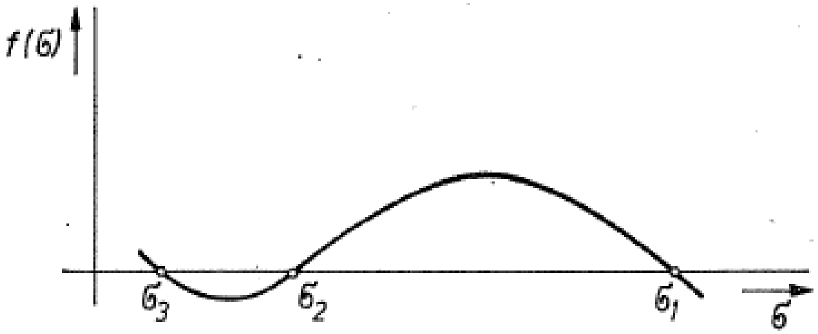
$$\begin{aligned}
&\sigma^3 - \sigma^2 (\sigma_x + \sigma_y + \sigma_z) + \sigma (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \\
&- \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2) - (\sigma_x \sigma_y \sigma_z + 2\tau_{xy} \tau_{yz} \tau_{zx} - \\
&- \sigma_x \tau_{yz}^2 - \sigma_y \tau_{zx}^2 - \sigma_z \tau_{xy}^2) = 0
\end{aligned}$$

Invarijante tenzora napona

$$\sigma^3 - \sigma^2(\sigma_x + \sigma_y + \sigma_z) + \sigma(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2) - (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2) = 0$$

$$\boxed{\sigma^3 - J_1\sigma^2 + J_2\sigma - J_3 = 0}$$

sva tri korena (rešenja) sekularne jednačine su prirodna i predstavljaju glavne napone (σ_1, σ_2, σ_3)



Pošto veličine glavnih normalnih napona ne zavise od orijentacije koordinatnog sistema, to ni koeficijenti kubne jednačine po σ (J_1 ; J_2 ; J_3) takođe od nje ne mogu zavisiti, zbog čega se nazivaju invarijante tenzora napona.

$$J_1(T_\sigma) \equiv \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3 = \text{const}^{*)}$$

$$J_2(T_\sigma) \equiv \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 = \text{const}.$$

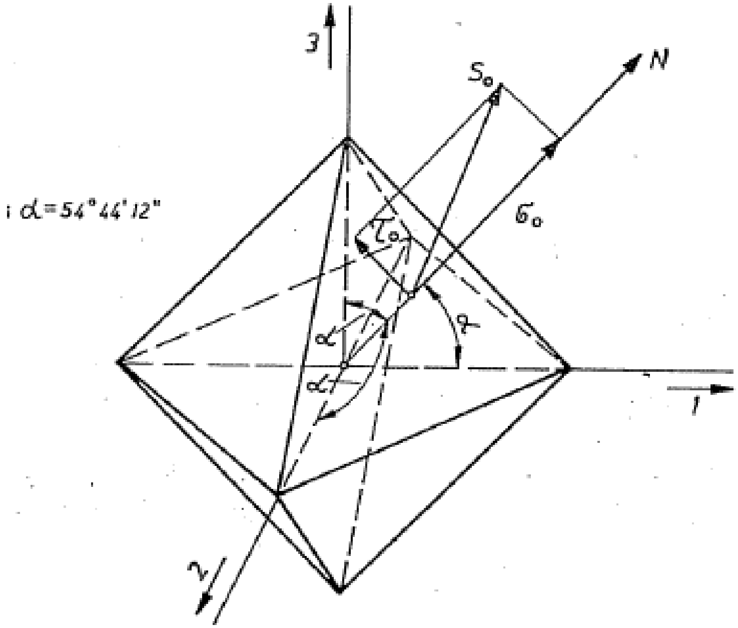
$$J_3(T_\sigma) \equiv \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix} = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 = \sigma_1\sigma_2\sigma_3 = \text{const}.$$

Oktaedarski naponi

Naponi koji deluju u ravnima koje su podjednako nagnute u odnosu na glavne ose (ima ih ukupno osam – formiraju oktaedar).

$$a_x^2 + a_y^2 + a_z^2 = 1$$

$$a_1 = a_2 = a_3 = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$$



normalni oktaedarski napon

$$\sigma_0 = \sigma_1 a_1^2 + \sigma_2 a_2^2 + \sigma_3 a_3^2 = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

smičući oktaedarski napon

$$\tau_0 = \sqrt{S_0^2 - \sigma_0^2} =$$

$$= \sqrt{\sigma_1^2 a_1^2 + \sigma_2^2 a_2^2 + \sigma_3^2 a_3^2 - (\sigma_1 a_1^2 + \sigma_2 a_2^2 + \sigma_3 a_3^2)^2} =$$

$$= \frac{\sqrt{2}}{3}\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

$$\tau_0 = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\tau_0 = \frac{2}{3}\sqrt{\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2}$$

$$\tau_0 = \frac{\sqrt{2J_1^2 - 6J_2}}{3} = \sqrt{\frac{2}{9}(J_1^2 - 3J_2)}$$

$$\tau_0 = \frac{1}{3}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

$$\boxed{\frac{2\sqrt{2}}{3} \approx 0{,}941 \geq \left|\frac{\tau_0}{\tau_{max}}\right| \geq \sqrt{\frac{2}{3}} \approx 0{,}816}$$

Srednji normalni napon (hidrostatički pritisak). Sferni tenzor napona

Srednji normalni napon ↔ oktaedarski normalni napon

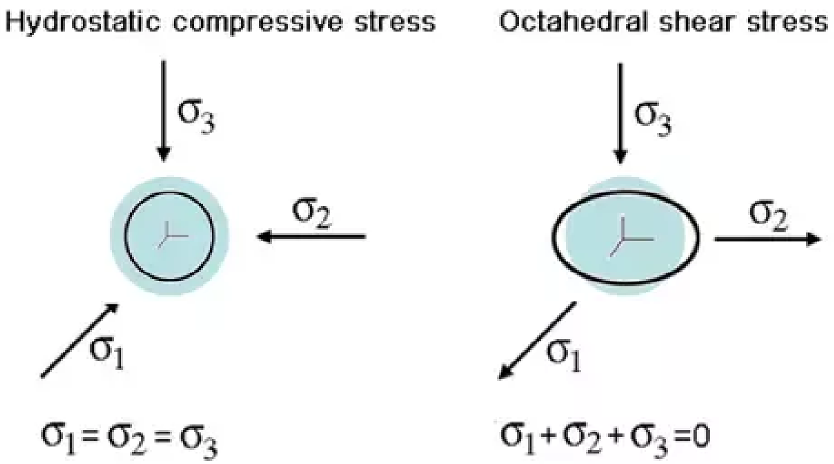
$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{J_1}{3}$$

Hidrostatički pritisak ili srednji pritisak

$$p = -\frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = -\sigma_m$$

Sferni tenzor napona $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_m.$

$$T_{S(\sigma)} = \begin{Bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{Bmatrix} = \sigma_m \{T_1\}$$



Jedinični tenzor

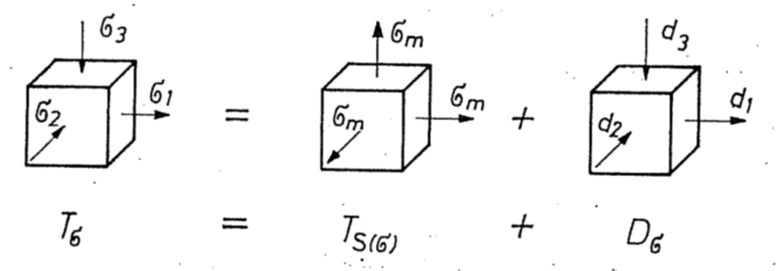
$$T_1 = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

Sferni tenzor napona ima dominantan uticaj na DEFORMABILNOST materijala!!!

Devijator napona

Razlaganje tenzora napona

$$T_\sigma = T_{S(\sigma)} + D_\sigma$$



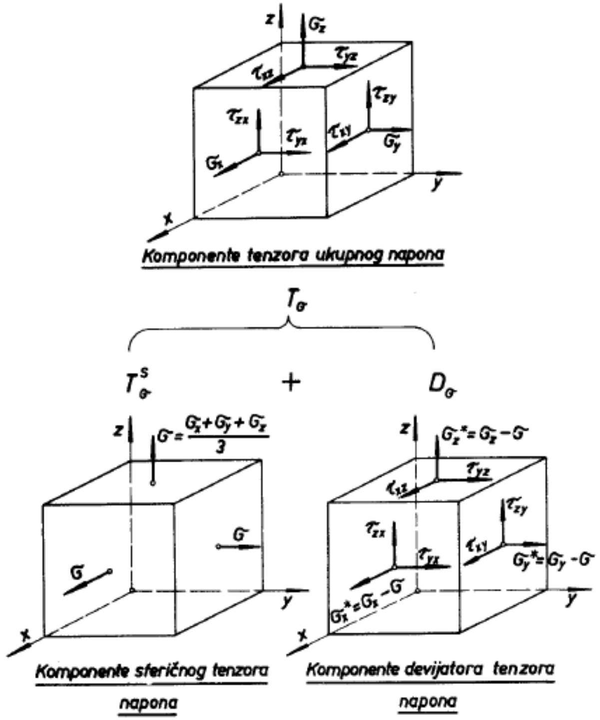
Devijator napona izražen preko komponenti napona

$$D_\sigma = T_\sigma - T_{S(\sigma)} =$$

$$= \begin{Bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{Bmatrix} - \begin{Bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{Bmatrix} =$$

$$= \begin{Bmatrix} (\sigma_x - \sigma_m) & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & (\sigma_y - \sigma_m) & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & (\sigma_z - \sigma_m) \end{Bmatrix} = \begin{Bmatrix} d_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & d_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & d_z \end{Bmatrix}$$

$$d_x = \sigma_x - \sigma_m \; ; \; d_y = \sigma_y - \sigma_m \; ; \; d_z = \sigma_z - \sigma_m .$$



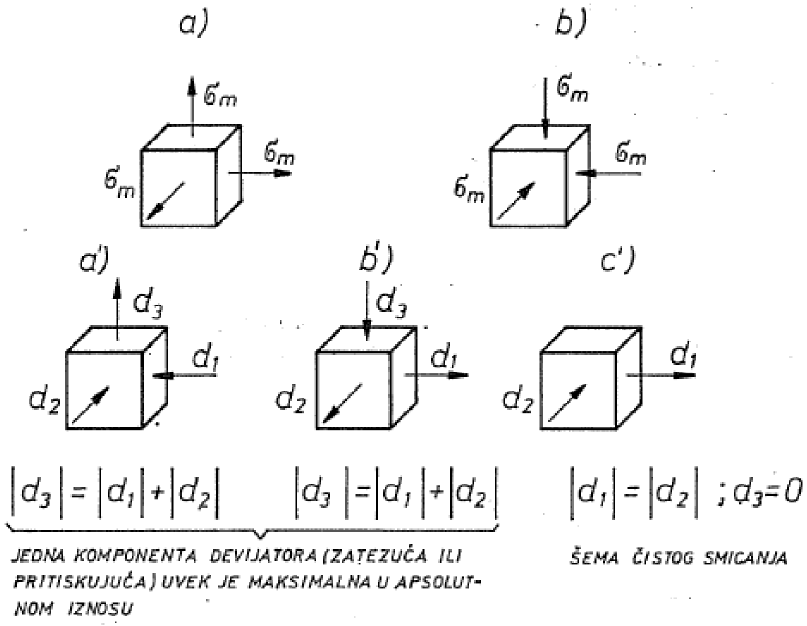
Devijator napona izražen preko glavnih napona

$$D_\sigma = \begin{Bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{Bmatrix} = \begin{Bmatrix} (\sigma_1 - \sigma_m) & 0 & 0 \\ 0 & (\sigma_2 - \sigma_m) & 0 \\ 0 & 0 & (\sigma_3 - \sigma_m) \end{Bmatrix}$$

$$d_x + d_y + d_z = d_1 + d_2 + d_3 = 0$$

Invarijante devijatora napona

Devijator napona – promena oblika
Sferični tenzor napona – promena zapremine



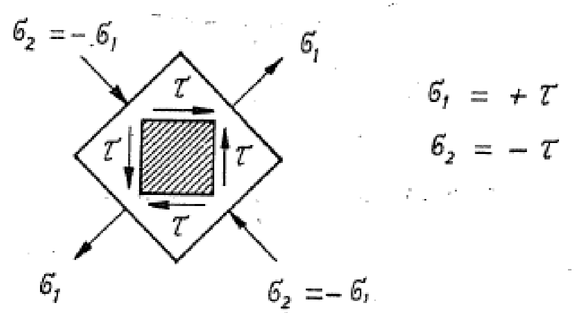
$$J_1(D_\sigma) \equiv (\sigma_x - \sigma_m) + (\sigma_y - \sigma_m) + (\sigma_z - \sigma_m) = 0$$

$$J_2(D_\sigma) \equiv (\sigma_x - \sigma_m)(\sigma_y - \sigma_m) + (\sigma_y - \sigma_m)(\sigma_z - \sigma_m) +$$

$$+ (\sigma_z - \sigma_m)(\sigma_x - \sigma_m) - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 =$$

$$= -\frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right]^{*)} - (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) =$$

$$= -\frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = d_1 d_2 + d_2 d_3 + d_3 d_1 = \text{const.}$$



Čisto smicanje (klizanje)

$$J_3(D_\sigma) \equiv \begin{vmatrix} d_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & d_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & d_z \end{vmatrix} = d_1 d_2 d_3 = \text{const.}$$

Efektivni (ekvivalentni) napon

Ekvivalentni normalni napona σ_e – skalarna invarijantna veličina

$$\sigma_e = \sqrt{3\left|J_2\left(D_\sigma\right)\right|} = \frac{\sqrt{2}}{2}\sqrt{\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]} =$$

$$= \frac{\sqrt{2}}{2}\sqrt{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2}$$

jednoosno naprezanje $\rightarrow \quad \sigma_e = \sigma_1$

Ekvivalentni smičući (tangencijalni) napona τ_e - skalarna invarijantna veličina

$$\tau_e = \frac{1}{\sqrt{6}}\sqrt{\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]} = \frac{1}{\sqrt{6}}\sqrt{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2}$$

$$\tau_e = \sqrt{\left|J_2\left(D_\sigma\right)\right|}$$

$$\tau_o = \frac{1}{3}\sqrt{\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]} = \frac{1}{3}\sqrt{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2}$$

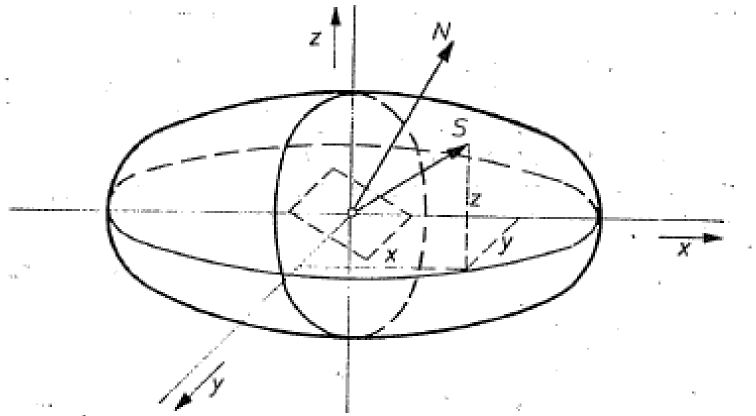
$$\boxed{\tau_e = \sqrt{\frac{3}{2}}\,\tau_o} \qquad \boxed{1{,}155 \approx \frac{2}{\sqrt{3}} \ge \frac{\tau_e}{\left|\tau_{max}\right|} \ge 1} \qquad \boxed{\frac{\tau_e}{\sigma_e} = \frac{\sqrt{3}}{3}}$$

Geometrijska interpretacija naponskih stanja. Elipsoid napona

Veličina ukupnog napona (S) na proizvoljno nagnutoj ravni određena je radijus vektorom tj. poluprečnikom elipsoida.

Dužine poluosa elepsoida – vrednosti glavnih napona.

$\sigma_1 = \sigma_2 = \sigma_3$ ellipsoid → sfera



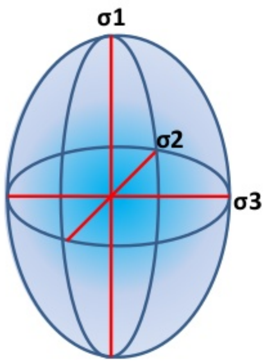
Jednačina elipsoida

$$S_1 = \sigma_1 a_1 \;;\quad S_2 = \sigma_2 a_2 \;;\quad S_3 = \sigma_3 a_3$$

$$a_1^2 = \frac{S_1^2}{\sigma_1^2} \;;\quad a_2^2 = \frac{S_2^2}{\sigma_2^2} \;;\quad a_3^2 = \frac{S_3^2}{\sigma_3^2}$$

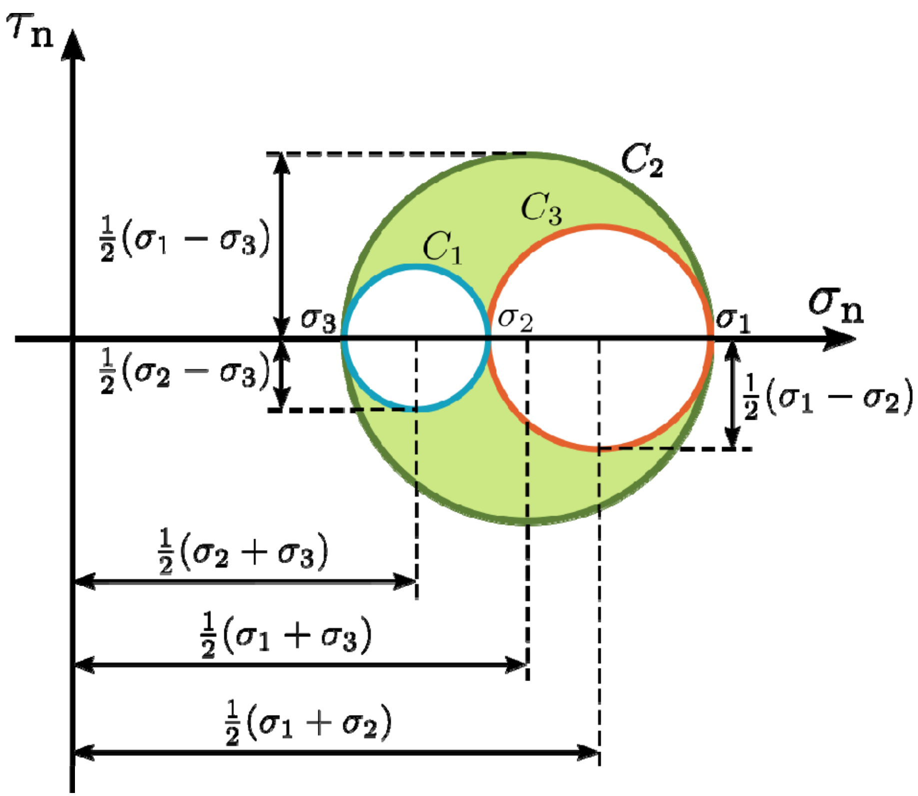
$$a_1^2 + a_2^2 + a_3^2 = 1$$

$$\boxed{\frac{S_1^2}{\sigma_1^2} + \frac{S_2^2}{\sigma_2^2} + \frac{S_3^2}{\sigma_3^2} = 1}$$



Morh-ovi krugovi napona

Mohrovi krugovi - dvodimenzionalni grafički prikaz vektora normalnog i tangencijalnog napona koje deluju na različito orijentisanim ravnima koje prolazi kroz posmatranu tačku.



Jednačine krugova

$$\sigma = \sigma_1 a_1^2 + \sigma_2 a_2^2 + \sigma_3 a_3^2$$

$$\tau^2 = S^2 - \sigma^2 = S_1^2 + S_2^2 + S_3^2 - \sigma^2$$

$$a_1^2 + a_2^2 + a_3^2 = 1$$

$$a_1^2 = \frac{\tau^2 + (\sigma - \sigma_2)(\sigma - \sigma_3)}{(\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3)}$$

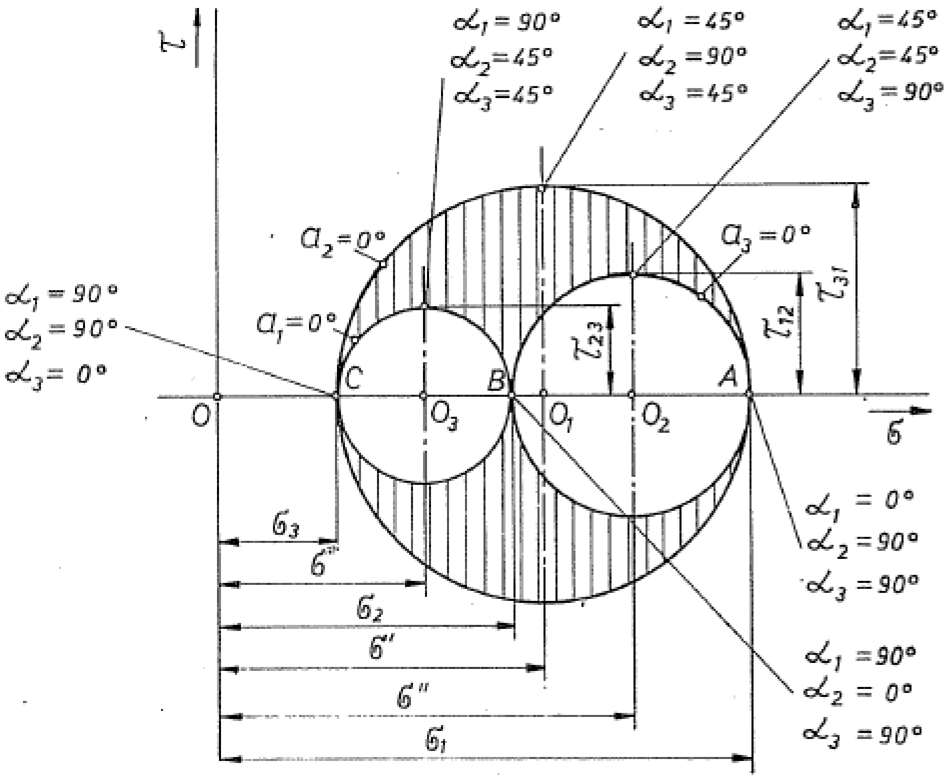
$$a_2^2 = \frac{\tau^2 + (\sigma - \sigma_3)(\sigma - \sigma_1)}{(\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)}$$

$$a_3^2 = \frac{\tau^2 + (\sigma - \sigma_1)(\sigma - \sigma_2)}{(\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2)}$$

Morh-ovi krugovi napona

$$(\sigma - \sigma')^2 + \tau^2 \le \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 \qquad (\sigma - \sigma'')^2 + \tau^2 \ge \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 \qquad (\sigma - \sigma''')^2 + \tau^2 \ge \left(\frac{\sigma_2 - \sigma_3}{2}\right)^2$$

$$\sigma' = \frac{\sigma_1 + \sigma_3}{2} \qquad \sigma'' = \frac{(\sigma_1 + \sigma_2)}{2} \qquad \sigma''' = \frac{\sigma_2 + \sigma_3}{2}$$



Smičući napon je maksimalan u ravni za koju važi:

$$\sigma = \frac{\sigma_1 + \sigma_3}{2}$$

Poluprečnici krugova :

$$|\tau_{12}| = \frac{\sigma_1 - \sigma_2}{2}$$

$$|\tau_{13}| = \frac{\sigma_1 - \sigma_3}{2}$$

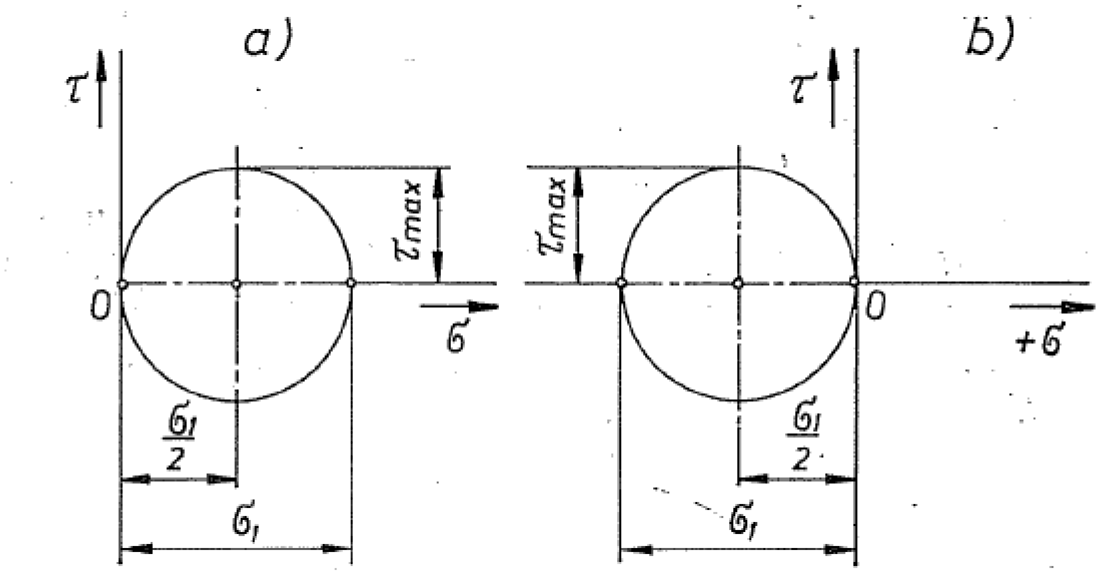
$$|\tau_{23}| = \frac{\sigma_2 - \sigma_3}{2}$$

Vrednosti glavnih napona u ravnima sa max. smičućim naponima :

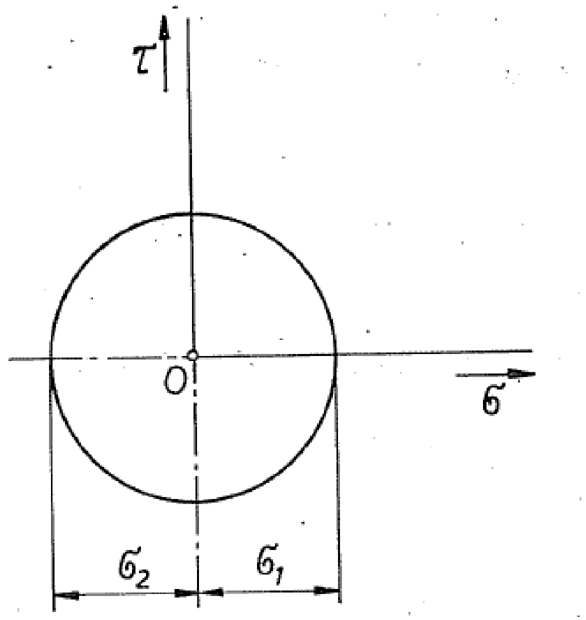
$$\frac{\sigma_2 + \sigma_3}{2}\,; \quad \frac{\sigma_3 + \sigma_1}{2}\,; \quad \frac{\sigma_1 + \sigma_2}{2}$$

Morh-ovi krugovi napona

Jednoosno naponsko stanje $\sigma_2=\sigma_3=0$:



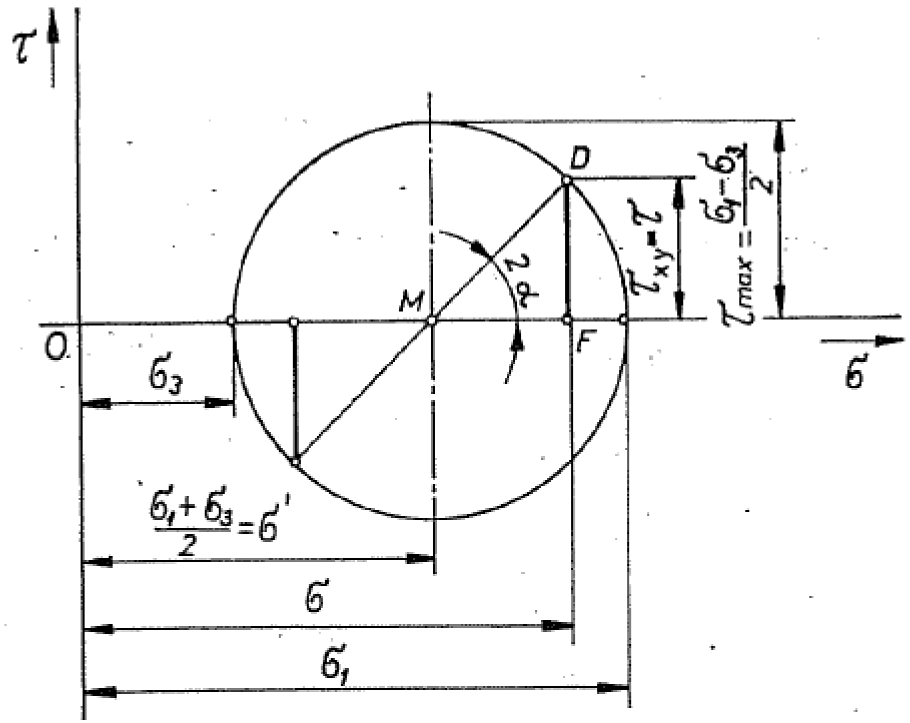
Mohr-ov krug za slučaj zatežućeg (a) i pritiskujućeg (b) jednoosnog naprezanja



Mohr-ov krug za slučaj čistog smicanja

Morh-ovi krugovi napona

Ravansko naponsko stanje $\sigma_2=0$:

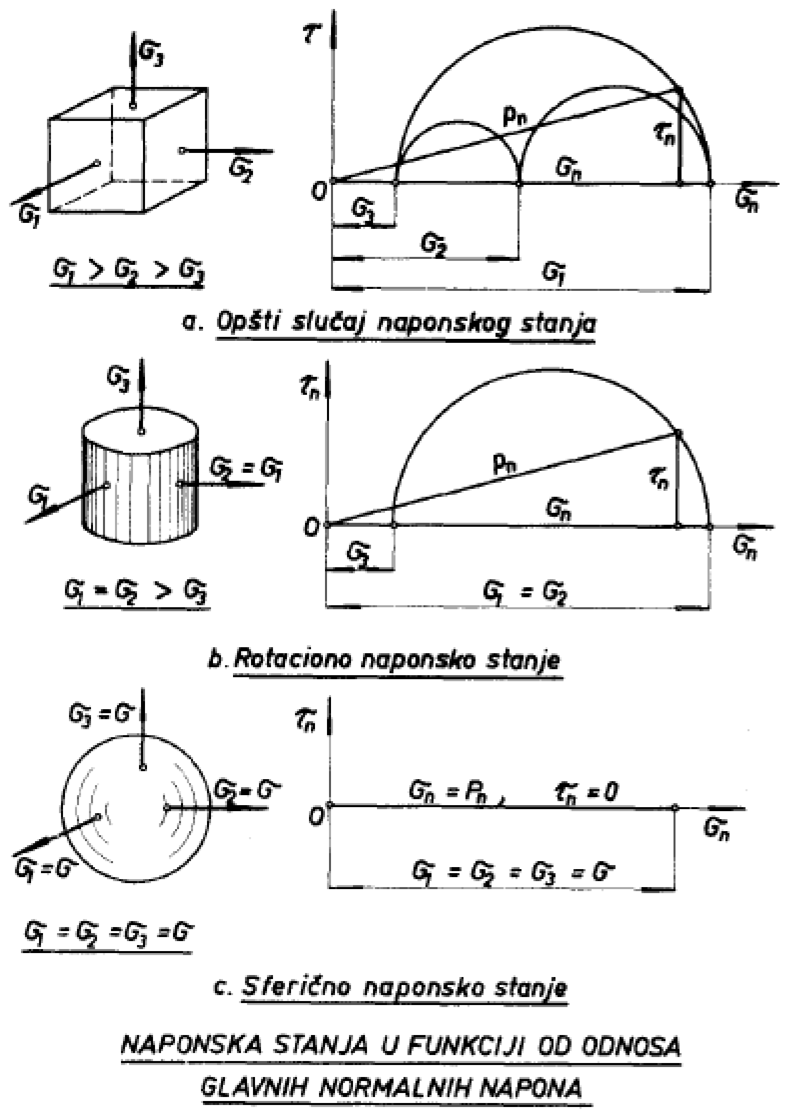


$$\tau_{max}^2 = \tau^2 + (\sigma - \sigma')^2$$

$$\overline{OF} = \sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\alpha = \sigma_1 \cos^2\alpha + \sigma_3 \sin^2\alpha$$

$$\overline{DF} = \tau = \frac{\sigma_1 - \sigma_3}{2}\sin 2\alpha$$

Morh-ovi krugovi napona

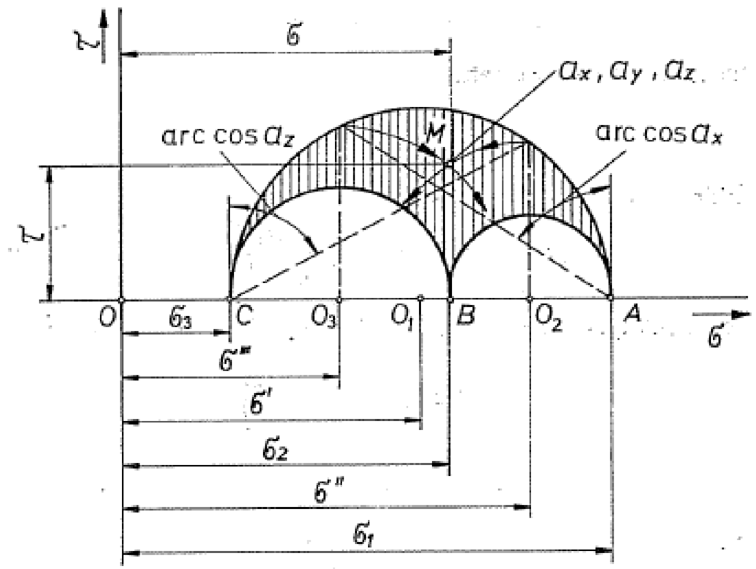


Koeficijent napona (Morh-Rozenberg-ov dijagram)

Koeficijent napona ν_σ

$$\nu_\sigma = \frac{\overline{O_1B}}{\overline{O_1A}} = \frac{\sigma_2 - \dfrac{\sigma_1 + \sigma_3}{2}}{\dfrac{\sigma_1 - \sigma_3}{2}} = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$$

$$\boxed{-1 \leqq \nu_\sigma \leqq 1}$$



$$\nu_\sigma = \frac{\sigma_{sr} - \dfrac{\sigma_{max} + \sigma_{min}}{2}}{\dfrac{\sigma_{max} - \sigma_{min}}{2}} = \frac{2\sigma_{sr} - \sigma_{max} - \sigma_{min}}{\sigma_{max} - \sigma_{min}}$$

$\sigma_1 = \sigma_2$ (tj. $\sigma_{sr} = \sigma_{max}$) : $\nu_\sigma = 1$

$\sigma_2 = \sigma_{sr} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{1}{2}(\sigma_{max} + \sigma_{min})$: $\nu_\sigma = 0$

$\sigma_2 = \sigma_3$ (tj. $\sigma_{sr} = \sigma_{min}$) : $\nu_\sigma = -1$

$$\sigma_2 = \nu_\sigma\left(\frac{\sigma_1 - \sigma_3}{2}\right) + \frac{\sigma_1 + \sigma_3}{2}$$

zatezanje $\rightarrow$ $-1 \le \nu_\sigma \le 0$

pritisak $\rightarrow$ $0 \le \nu_\sigma \le 1$

čisto smicanje (uvijanje) $\rightarrow$ $\nu_\sigma = 0$

Ugao naponskog stanja

Koeficijent C_σ

$$|\tau_{13}| = |\tau_{max}| = \frac{|\sigma_1 - \sigma_3|}{2}$$

$$\sigma_1 - \sigma_3 = 2|\tau_{max}|$$

$$\tau_e = \frac{1}{\sqrt{6}}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} =$$

$$= \frac{1}{\sqrt{6}}\,\frac{2}{\sqrt{2}}\,|\tau_{max}|\,\sqrt{\nu_\sigma^2 + 3}$$

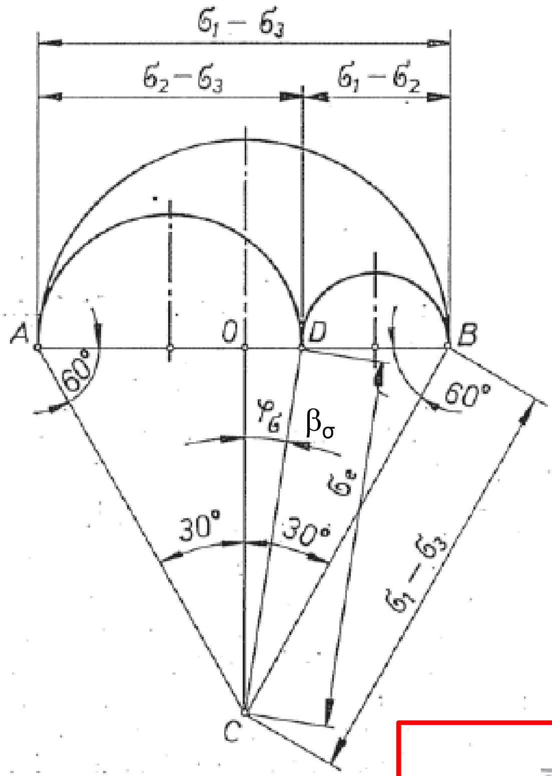
$$|\tau_{max}| = \frac{\tau_e\sqrt{3}}{\sqrt{\nu_\sigma^2 + 3}}$$

$$C_\sigma = \frac{\tau_e}{|\tau_{max}|} = \sqrt{\frac{\nu_\sigma^2}{3} + 1}$$

$$1{,}155 \geq C_\sigma \geq 1$$

$$0{,}941 \approx \frac{2\sqrt{2}}{3} \geq \frac{|\tau_0|}{|\tau_{max}|} \geq \sqrt{\frac{2}{3}} \approx 0{,}816$$

Ugao naponskog stanja $\varphi_\sigma=\beta_\sigma$



$$\operatorname{tg}\varphi_\sigma = \frac{\overline{OD}}{\overline{OC}} = \frac{\sigma_2 - \frac{\sigma_1+\sigma_3}{2}}{\sqrt{3}\,\frac{\sigma_1-\sigma_3}{2}} = \frac{\nu_\sigma}{\sqrt{3}}$$

$$\overline{OA} = \overline{OB} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\overline{OC} = \frac{\sqrt{3}}{2}(\sigma_1 - \sigma_3)$$

$$\overline{DC} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} = \sigma_e$$

$$\overline{OD} = \sigma_2 - \frac{\sigma_1+\sigma_3}{2}$$

Vidovi naponskih stanja

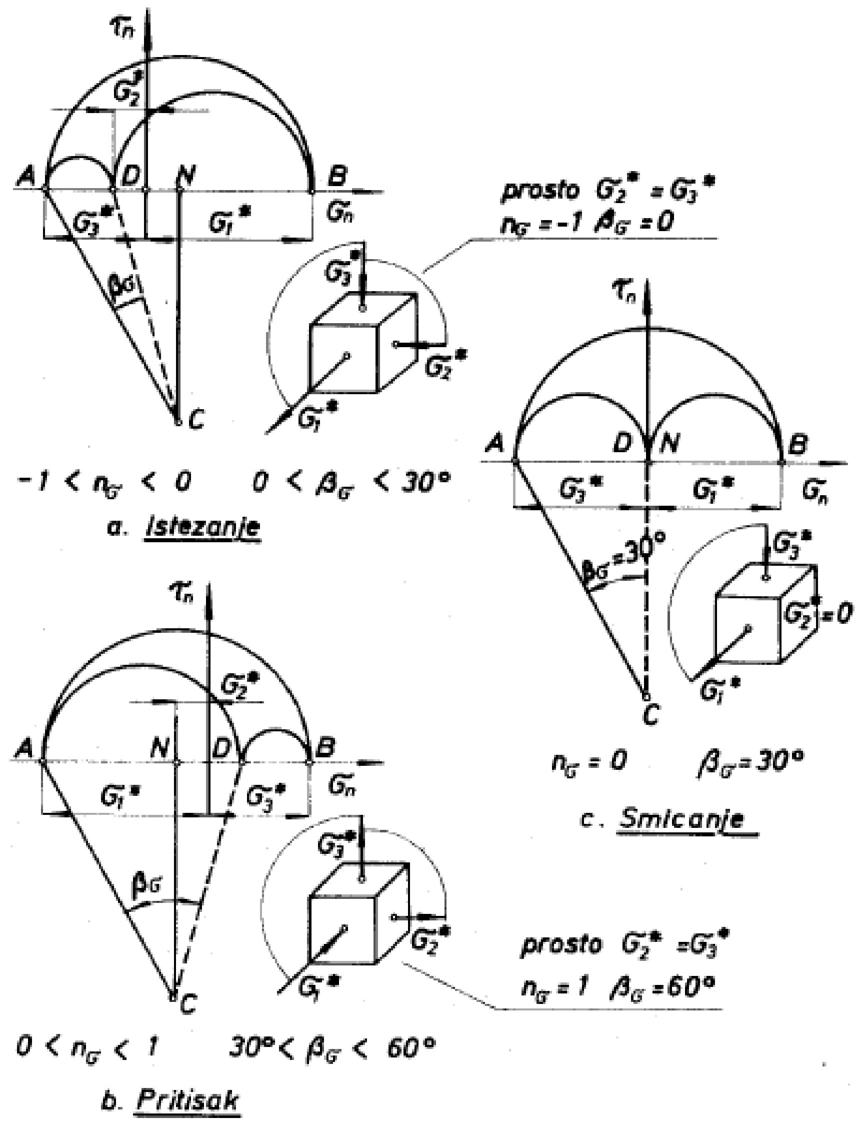
$$\nu_\sigma = n_\sigma$$

$$\varphi_\sigma = \beta_\sigma$$

zatezanje $\rightarrow$ $-1 \leq \nu_\sigma \leq 0$

pritisak $\rightarrow$ $0 \leq \nu_\sigma \leq 1$

čisto smicanje (uvijanje) $\rightarrow$ $\nu_\sigma = 0$



VIDOVI NAPONSKIH STANJA NA OSNOVU GLAVNIH KOMPONENATA DEVIJATORA TENZORA NAPONA